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Dyscalculia: From Brain to Education

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Recent research in cognitive and developmental neuroscience is providing a new approach to the understanding of dyscalculia that emphasizes a core deficit in understanding sets and their numerosities, which is fundamental to all aspects of elementary school mathematics. The neural bases of numerosity processing have been investigated in structural and functional neuroimaging studies of adults and children, and neural markers of its impairment in dyscalculia have been identified. New interventions to strengthen numerosity processing, including adaptive software, promise effective evidence-based education for dyscalculic learners.

Developmental dyscalculia is a mathematical disorder, with an estimated prevalence of about 5 to 7% (*1*), which is roughly the same prevalence as developmental dyslexia (*2*). A major report by the UK government concludes, "Developmental dyscalculia is currently the poor relation of dyslexia, with a much lower public profile. But the consequences of dyscalculia are at least as severe as those for dyslexia" [(*3*), p. 1060]. The relative poverty of dyscalculia funding is clear from the figures: Since 2000, NIH has spent $107.2 million funding dyslexia research but only $2.3 million on dyscalculia (*4*).

The classical understanding of dyscalculia as a clinical syndrome uses low achievement on mathematical achievement tests as the criterion without identifying the underlying cognitive phenotype (*5*–*7*). It has therefore been unable to inform pathways to remediation, whether in focused interventions or in the larger, more complex context of the math classroom.

Why Is Mathematical Disability Important?

Low numeracy is a substantial cost to nations, and improving standards could dramatically improve economic performance. In a recent analysis, the Organisation for Economic Co-operation and Development (OECD) demonstrated that an improvement of "one-half standard deviation in mathematics and science performance at the individual level implies, by historical experience, an increase in annual growth rates of GDP per capita of 0.87%" [(*8*), p. 17]. Time-lagged correlations show that improvements in educational performance contribute to increased GDP growth. A substantial long-term improvement in GDP growth (an added 0.68% per annum for all OECD countries) could be achieved just by raising the standard of the lowest-attaining students to the Programme for International Student Assessment (PISA) minimum level (Box 1). In the United States, for example, this would mean bringing the lowest 19.4% up to the minimum level, with a corresponding 0.74% increase in GDP growth.

Besides reduced GDP growth, low numeracy is a substantial financial cost to governments and personal cost to individuals. A large UK cohort study found that low numeracy was more of a handicap for an individual's life chances than low literacy: They earn less, spend less, are more likely to be sick, are more likely to be in trouble with the law, and need more help in school (*9*). It has been estimated that the annual cost to the UK of low numeracy is £2.4 billion (*10*).

What Is Dyscalculia?

Recent neurobehavioral and genetic research suggests that dyscalculia is a coherent syndrome that reflects a single core deficit. Although the literature is riddled with different terminologies, all seem to refer to the existence of a severe disability in learning arithmetic. The disability can be highly selective, affecting learners with normal intelligence and normal working memory (*11*), although it co-occurs with other developmental disorders, including reading disorders (*5*) and attention deficit hyperactivity disorder (ADHD) (*12*) more often than would be expected by chance. There are high-functioning adults who are severely dyscalculic but very good at geometry, using statistics packages, and doing degree-level computer programming (*13*).

There is evidence that mathematical abilities have high specific heritability. A multivariate genetic analysis of a sample of 1500 pairs of monozygotic and 1375 pairs of dizygotic 7-year-old twins found that about 30% of the genetic variance was specific to mathematics (*14*). Although there is a significant co-occurrence of dyscalculia with dyslexia, a study of first-degree relatives of dyslexic probands revealed that numerical abilities constituted a separate factor, with reading-related and naming-related abilities being the two other principal components (*15*). These findings imply that arithmetical learning is at least partly based on a cognitive system that is distinct from those underpinning scholastic attainment more generally.

This genetic research is supported by neurobehavioral research that identifies the representation of numerosities—the number of objects in a

Box 1: PISA question example

At Level 1, students can answer questions involving familiar contexts in which all relevant information is present and the questions are clearly defined. They are able to identify information and carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli. For example:

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was 1 SGD = 4.2 ZAR

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

79% of 15-year-olds were able to answer this correctly.

Box 2: Dyscalculia observed

Examples of common indicators of dyscalculia are (i) carrying out simple number comparison and addition tasks by counting, often using fingers, well beyond the age when it is normal, and (ii) finding approximate estimation tasks difficult. Individuals identified as dyscalculic behave differently from their mainstream peers. For example:

To say which is the larger of two playing cards showing 5 and 8, they count all the symbols on each card.

To place a playing card of 8 in sequence between a 3 and a 9, they count up spaces between the two to identify where the 8 should be placed.

To count down from 10, they count up from 1 to 10, then 1 to 9, etc.

To count up from 70 in tens, they say "70, 80, 90, 100, 200, 300…"

They estimate the height of a normal room as "200 feet?"

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set—as a foundational capacity in the development of arithmetic (*16*). This capacity is impaired in dyscalculic learners even in tasks as simple as enumerating small sets of objects (*11*) or comparing the numerosities of two arrays of dots (*17*). The ability to compare dot arrays has been correlated with more general arithmetical abilities both in children (*18*) and across the age range (*17*, *19*). This core deficit in processing numerosities is analogous to the core deficit in phonological awareness in dyslexia (Box 2) (*2*).

Although there is little longitudinal evidence, it seems that dyscalculia persists into adulthood (*20*), even among individuals who are able in other cognitive domains (*13*). The effects of early and appropriate intervention with dyscalculia have yet to be investigated. This also leaves open the question as to whether there is a form of dyscalculia that is a delay, rather than a deficit, that will resolve, perhaps with appropriate educational support.

Converging evidence of dyscalculia as a distinct deficit comes from studies of impairments in the mental and neural representation of fingers. It has been known for many years that fingers are used in acquiring arithmetical competence (*21*). This involves understanding the mapping between the set of fingers and the set of objects to be enumerated. If the mental representation of fingers is weak, or if there is a deficit in understanding the numerosity of sets, then the child's cognitive development may fail to establish the link between fingers and numerosities. In fact, developmental weakness in finger representation ("finger agnosia") is a predictor of arithmetical ability (*22*). Gerstmann's Syndrome, whose symptoms include dyscalculia and finger agnosia, is due to an abnormality in the parietal lobe and, in its developmental form, is also associated with poor arithmetical attainment (*23*).

Numbers do not seem to be meaningful for dyscalculics—at least, not meaningful in the way that they are for typically developing learners. They do not intuitively grasp the size of a number and its value relative to other numbers. This basic understanding underpins all work with numbers and their relationships to one another.

What Do We Know About the Brain and Mathematics?

The neural basis of arithmetical abilities in the parietal lobes, which is separate from language and domain-general cognitive capacities, has been broadly understood for nearly 100 years from research on neurological patients (*24*). One particularly interesting finding is that arithmetical concepts and laws can be preserved even when facts have been lost (*25*), and conversely, facts can be preserved even when an understanding of concepts and laws has been lost (*26*).

Neuroimaging experiments confirm this picture and show links from the parietal lobes to the left frontal lobe for more complex tasks (*27*, *28*). One important new finding is that the neural organization of arithmetic is dynamic, shifting from one subnetwork to another during the process of learning. Thus, learning new arithmetical facts primarily involves the frontal lobes and the intraparietal sulci (IPS), but using previously learned facts involves the left angular gyrus, which is also implicated in retrieving facts from memory (*29*). Some of the principal links are summarized in Fig. 1. Even prodigious calculators use this network, although supplementing it with additional brain areas (*30*) that appear to extend the capacity of working memory (*31*).

There is now extensive evidence that the IPS supports the representation of the magnitude of symbolic numbers (*32*, *33*), either as analog magnitudes or as a discrete representation that codes cardinality, as evidenced by IPS activation when processing the numerosity of arrays of objects (*34*). Moreover, when IPS functioning is disturbed by magnetic stimulation, the ability to estimate discrete magnitudes is affected (*35*, *36*). The critical point is that almost all arithmetical and numerical processes implicate the parietal lobes, especially the IPS, suggesting that these are at the core of mathematical capacities.

Patterns of brain activity in 4-year-olds and adults show overlapping areas in the parietal lobes bilaterally when responding to changes in numerosity (*37*). Nevertheless, there is a developmental trajectory in the organization of more complex arithmetical abilities. First, the organization of routine numerical activity changes with age, shifting from frontal areas (which are associated with executive function and working memory) and medial temporal areas (which are associated with declarative memory) to parietal areas (which are associated with magnitude processing and arithmetic fact retrieval) and occipito-temporal areas (which are associated with processing symbolic form) (*38*). These changes allow the brain to process numbers more efficiently and automatically, which enables it to carry out the more complex processing of arithmetical calculations. As A. N. Whitehead observed, an understanding of symbolic notation relieves "the brain of all unnecessary work … and sets it free to concentrate on more advanced problems" (*39*).

This suggests the possibility that the neural specialization for arithmetical processing may arise, at least in part, from a developmental interaction between the brain and experience (*40*, *41*). Thus, one way of thinking about dyscalculia is that the typical school environment does not provide the right kind of experiences to enable the dyscalculic brain to develop normally to learn arithmetic.

Of course, mathematics is more than just simple number processing and retrieval of previously learned facts. In a numerate society, we have to

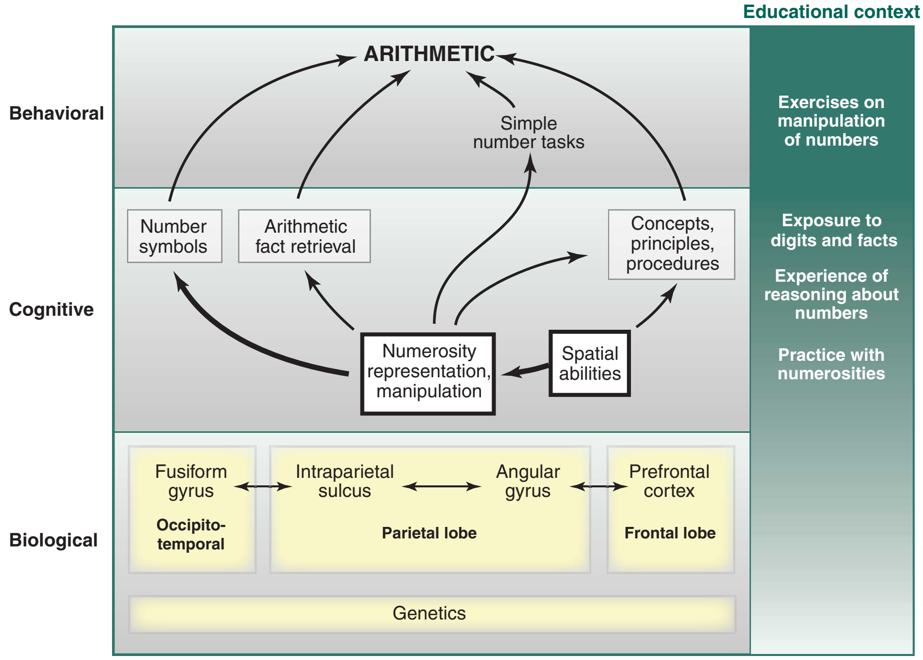

Fig. 1. Causal model of possible inter-relations between biological, cognitive, and simple behavioral levels. Here, the only environmental factors we address are educational. If parietal areas, especially the IPS, fail to develop normally, there will be an impairment at the cognitive level in numerosity representation and consequential impairments for other relevant cognitive systems revealed in behavioral abnormalities. The link between the occipitotemporal and parietal cortex is required for mapping number symbols (digits and number words) to numerosity representations. Prefrontal cortex supports learning new facts and procedures. The multiple levels of the theory suggest the instructional interventions on which educational scientists should focus.

learn more complex mathematical concepts, such as place value, and more complex procedures, such as "long" addition, subtraction, multiplication, and division. Recent research has revealed the neural correlates of learning to solve complex, multidigit arithmetic problems (*24*). Again, this research shows that solving new problems requires more activation in the inferior frontal gyrus for reasoning and working memory and the IPS for representing the magnitudes of the numbers involved, as compared with retrieval of previously learned facts (*42*).

The striking result in all of these studies is the crucial role of the parietal lobes. That the IPS is implicated in both simple and complex calculations suggests that the basic representations of magnitude are always activated, even in the retrieval of well-learned single-digit addition and multiplication facts (*43*). This is consistent with the well-established "problem-size effect," in which single-digit problems take longer to solve the larger the operands, even when they are well known (*44*). It seems that the typically developing individual, even when retrieving math facts from memory, cannot help but activate the meaning of the component numbers at the same time. If that link has not been established, calculation is necessarily impaired.

What Do We Know About the Brain and Dyscalculia?

The clinical approach has identified behavioral deficits in dyscalculic learners typically by performance on standardized tests of arithmetic. However, even in primary school (K1 to 5), arithmetical competence involves a wide range of cognitive skills, impairments in any of which may affect performance, including reasoning, working memory, language understanding, and spatial cognition (*45*). Neural differences in structure or functioning may reflect differences in any of these cognitive skills. However, if dyscalculia is a core deficit in processing numerosities, then abnormalities should be found in the parietal network that supports the enumeration of small sets of objects (*11*) and the comparison of numerosities of arrays of dots (*17*).

The existence of a core deficit in processing numerosities is consistent with recent discoveries about dyscalculic brains: (i) Reduced activation has been observed in children with dyscalculia during comparison of numerosities (*46*, *47*), comparison of number symbols (*46*), and arithmetic (*48*)—these children are not using the IPS so much during these tasks. (ii) Reduced gray matter in dyscalculic learners has been observed in areas known to be involved in basic numerical processing, including the left IPS (*49*), the right IPS (*50*), and the IPS bilaterally (*51*)—these learners have not developed these brain areas as much as typical learners (Fig. 2). And (iii) differences in connectivity among the relevant parietal regions, and between these parietal regions and occipitotemporal regions associated with processing symbolic number form (Fig. 2), are revealed through diffusion tensor imaging tractography (*51*)—dyscalculic learners have not sufficiently developed the structures needed to coordinate the components needed for calculation.

Figure 1 suggests that the IPS is just one component in a large-scale cortical network that subserves mathematical cognition. This network may break down in multiple ways, leading to different dyscalculia patterns of presenting symptoms when the core deficit is combined with other cognitive deficits, including working memory, reasoning, or language (*52*).

Moreover, the same presenting symptom could reflect different impairments in the network. For example, abnormal comparison of symbolic numbers (*53*, *54*) could arise from an impairment in the fusiform gyrus associated with visual processing of number symbols, an impairment in IPS associated with the magnitude referents, analog or numerosity, of the number symbols, or reduced connectivity between fusiform gyrus and IPS.

How Is This Relevant to Maths Education?

There have been many attempts to raise the performance of children with low-numeracy, although not specifically dyscalculia. In the United States, evidence-based approaches have focused on children from deprived backgrounds, usually low socioeconomic status (*46*, *47*). The current National Strategy in the United Kingdom gives special attention to children with low numeracy by (i) diagnosing each child's conceptual gaps in understanding and (ii) giving the child more individual support in working through visual, verbal, and physical activities designed to bridge each gap. Unfortunately, there is still little quantitative evaluation of the effectiveness of the strategy since it was first piloted in 2003: "the evaluations used very diverse measures; and most did not include ratio gains or effect sizes or data from which these could be obtained" [(*55*), p. 13].

It has not been possible to tell, therefore, whether identifying and filling an individual's conceptual gaps with a more individualized version of the same teaching is effective. A further problem is that these interventions are effective when there has been specialist training for teaching assistants, but not all schools can provide this (*55*).

These standardized approaches depend on curriculum-based definitions of typical arithmetical development, and how children with low numeracy differ from the typical trajectory. In contrast, neuroscience research suggests that rather than address isolated conceptual gaps, remediation should build the foundational number concepts first. It offers a clear cognitive target for assessment and intervention that is largely independent of the learners' social and educational circumstances. In the assessment of individual cognitive capacities, set enumeration and comparison can supplement performance on curriculum-based standardized tests of arithmetic to differentiate dyscalculia from other causes of low numeracy (*11*, *56*, *57*).

In intervention, strengthening the meaningfulness of numbers, especially the link between the math facts and their component meanings, is crucial. As noted above, typical retrieval of simple arithmetical facts from memory elicits activation of the numerical value of the component numbers.

Without specialized intervention, most dyscalculic learners are still struggling with basic arithmetic in secondary school (*20*). Effective early intervention may help to reduce the later impact on poor numeracy skills, as it does in dyslexia (*58*). Although very expensive, it promises to repay 12 to 19 times the investment (*10*).

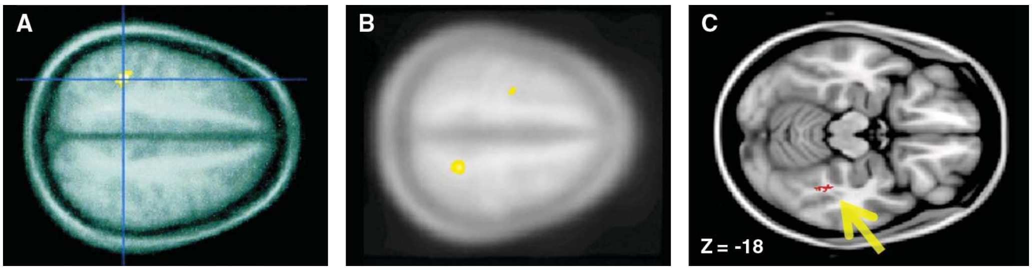

Fig. 2. Structural abnormalities in young dyscalculic brains suggesting the critical role for the IPS. Here, we show areas where the dyscalculic brain is different from that of typically developing controls. Both left and right IPS are implicated, possibly with a greater impairment for left IPS in older learners. (**A**) There is a small region of reduced gray-matter density in left IPS in adolescent dyscalculics (*41*). (**B**) There is right IPS reduced gray-matter density (yellow area) in 9-year-olds (*42*). (**C**) There is reduced probability of connections from right fusiform gyrus to other parts of the brain, including the parietal lobes (*43*).

What Can Be Done?

Although the neuroscience may suggest what should be taught, it does not specify how it should be taught. Concrete manipulation activities have been used for many decades in math remediation because they provide tasks that make number concepts meaningful (*59*), providing an intrinsic relationship between a goal, the learner's action, and the informational feedback on the action (*60*). Educators recognize that informational feedback provides intrinsic motivation in a task, and that is of greater value to the learner than the extrinsic motives and rewards provided by a supervising teacher (*61–63*).

Experienced special educational needs (SEN) teachers use these activities in the form of games with physical manipulables (such as Cuisenaire rods, number tracks, and playing cards) to give learners experience of the meaning of number. Through playing these games, learners can discover from their manipulations, for example, which rod fits with an 8-rod to match a 10-rod. However, these methods require specially trained teachers working with a single learner or a small group of learners (*64–66*) and are allotted only limited time periods in the school schedule.

A promising approach, therefore, is to construct adaptive software informed by the neuroscience findings on the core deficit in dyscalculia. Such software has the potential to reduce the demand on specially trained teachers and to transcend the limits of the school schedule. There have been two examples of adaptive games based on neuroscience findings.

The *Number Race* (*67*) targets the inherited approximate numerosity system in the IPS (*68*) that may support early arithmetic (*18*). In dyscalculics, this system is less precise (*17*), and the training is designed to improve its precision. The task is to select the larger of two arrays of dots, and the software adapts to the learner, making the difference between the arrays smaller as their performance improves, and provides informational feedback as to which is correct.

Another adaptive game, *Graphogame-Maths*, targets the inherited system for representing and manipulating sets in the IPS, which is impaired in dyscalculia (*45*). Again, the basis of the game is the comparison of visual arrays of objects, but here the sets are small and can be counted, and the progressive tasks are to identify the link between the number of objects in the sets and their verbal numerical label, with informational feedback showing which is correct.

The effectiveness of the two games was compared in a carefully controlled study of kindergarten children (aged 6 to 7 years) who were identified by their teachers as needing special support in early maths. After 10 to 15 min of play per school day for 3 weeks, there was a significant improvement in the task practiced in both games—namely, number comparison—but the effect did not generalize to counting or arithmetic. *Graphogame-Maths* appeared to lead to slightly better and longer-lasting improvement in number comparison (*69*).

Although the *Number Race* and *Graphogame-Maths* are adaptive games based on neuroscience findings, neither requires learners to manipulate numerical quantities. Manipulation is critical for providing an intrinsic relationship between task goals, a learner's actions, and informational feedback on those actions (*60*). When a learning environment provides informational feedback, it enables the learner to work out how to adjust their actions in relation to the goal, and they can be their own "critic," not relying on the teacher to guide (*61–63*). This is analogous to the "actor-critic" model of unsupervised reinforcement learning in neuroscience, which proposes a critic element internal to the learning mechanism—not a guide that is external to it—which evaluates the informational feedback in order to construct the next action (*70*).

A different approach, one that emulates the manipulative tasks used by SEN teachers, has been taken in adaptive software that, driven by the neuroscience research on dyscalculia, focuses on numerosity processing (Fig. 3) (*56*). The informational feedback here is not an external critic showing the correct answer. Rather, the visual representation of two rods that match a given distance, or not, enables the learner to interpret for themselves what the improved action should be—and can serve as their own internal critic.

An additional advantage of adaptive software is that learners can do more practice per unit time than with a teacher. It was found that for "SEN learners (12-year-olds) using the Number Bonds game [illustrated in Fig. 3], 4–11 trials per minute were completed, while in an SEN class of three supervised learners only 1.4 trials per minute were completed during a 10-min observation" [(*56*), p. 535]. In another SEN group of 11 year olds, the game elicited on average 173 learner manipulations in 13 min (where a perfect performance, in which every answer is correct, is 88 in 5 min because the software adapts the timing according to the response). In this way, neuroscience research is informing what should be targeted in the next generation of adaptive software.

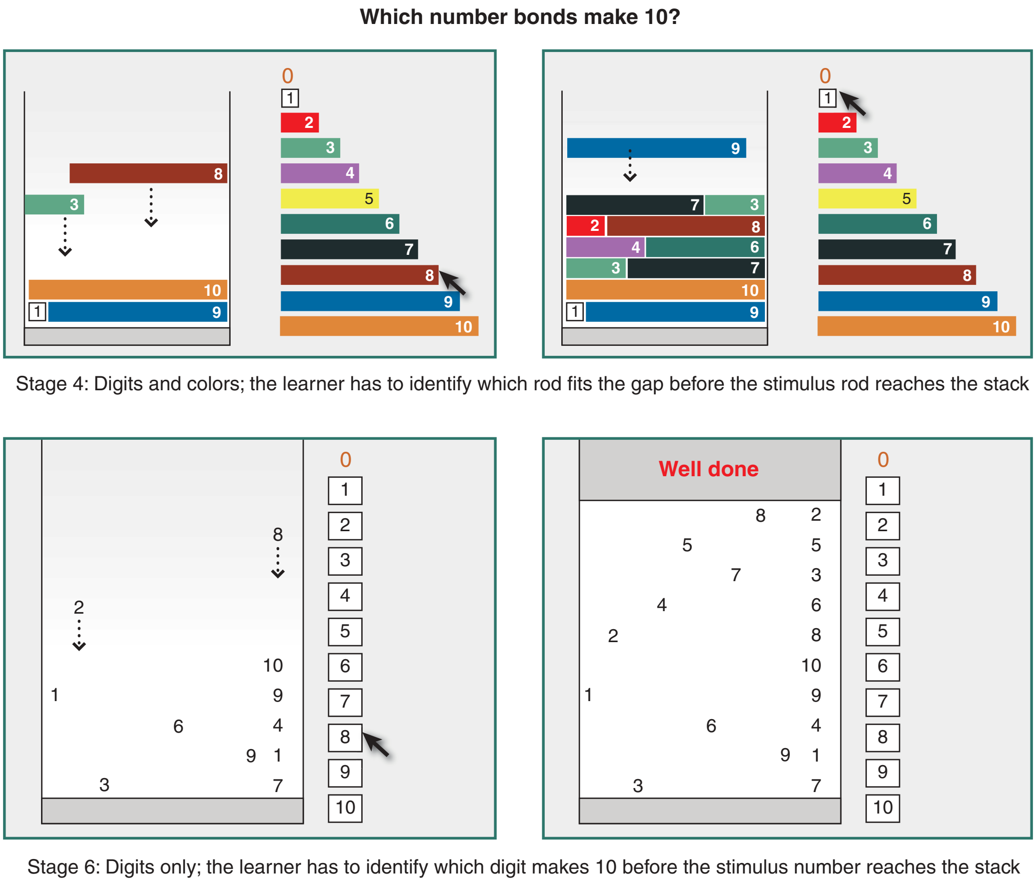

Fig. 3. Remediation using learning technology. The images are taken from an example of an interactive, adaptive game designed to help the learner make the link between digits and their meaning. The timed version of the number bonds game elicits many learner actions with informational feedback, and scaffolds abstraction, through stages 1, 5 colors + lengths, evens; 2, 5 colors + lengths, odds; 3, all 10 colors + lengths; 4, digits + colors + lengths; 5, digits + lengths; and 6, digits only. Each rod falls at a pace adapted to the learner's performance, and the learner has to click the corresponding rod or number to make 10 before it reaches the stack (initially 3 s); if there is a gap, or overlap, or they are too slow, the rods dissolve. When a stack is complete, the game moves to the next stage. The game is available from www.number-sense.co.uk/numberbonds/.

At present, it is not yet clear whether early and appropriately targeted interventions can turn a dyscalculic into a typical calculator. Dyscalculia may be like dyslexia in that early intervention can improve practical effectiveness without making the cognitive processing like that of the typically developing.

What Is the Outlook?

Recent research by cognitive and developmental scientists is providing a scientific characterization of dyscalculia as reduced ability for understanding numerosities and mapping number symbols to number magnitudes. Personalized learning applications developed by educational scientists can be targeted to remediate these deficits and can be implemented on handheld devices for independent learning. Because there are also individual differences in numerosity processing in the normal range (*17*, *19*, *71*), the same programs can assist beginning mainstream learners, so one can envisage a future in which all learners will benefit from these developments.

Although much progress has been made, a number of open questions remain.

(i) The possible existence of a variety of dyscalculia behavioral patterns of impairment raises the interesting question of whether one deficit—numerosity estimation—is a necessary or sufficient for a diagnosis of dyscalculia. This is critical because it has implications for whether a single diagnostic assessment based on numerosity is sufficient, or whether multiple assessments are required. It also has implications for whether numerosity should be the focus of remediation, or whether other (perhaps more symbolic) activities must also be targeted.

(ii) Does the sensitivity of neuroscience measures make it possible to identify learners at risk for dyscalculia earlier than is possible with behavioral assessments, as is the case with dyslexia (*72*)?

(iii) Further research is needed on the neural consequences of intervention. Even where intervention improves performance, it may not be clear whether the learner's cognitive and neural functioning has become more typical or whether compensatory mechanisms have developed. This would require more extensive research, as in the case of dyslexia, in which functional neuroimaging has revealed the effects of successful behavioral interventions on patterns of neural function (*2*).

(iv) Personalized learning applications enable fine-grain evaluation of theory-based instructional interventions. For example, the value of active manipulation versus associative learning on one hand, and of intrinsic versus extrinsic motivation and feedback on the other, can be assessed by orthogonally manipulating these features of learning environments. Their effects can be assessed behaviorally in terms of performance on the tasks by target learners. Their effects can also be assessed by investigating neural changes over time in target learners, through structural and functional neuroimaging. Classroom-based but theory-based testing holds great potential for the development of educational theory and could contribute in turn to testing hypotheses in neuroscience.

(v) At the moment, dyscalculia is not widely recognized by teachers or educational authorities nor, it would seem, by research-funding agencies. Recognition is likely to be the basis for improved prospects for dyscalculic sufferers.

There is an urgent societal need to help failing learners achieve a level of numeracy at which they can function adequately in the modern workplace. Contemporary research on dyscalculia promises a productive way forward, but it is still a "poor relation" in terms of funding (*4*), which means there is a serious lack of evidence-based approaches to dyscalculia intervention. An understanding of how the brain processes underlying number and arithmetic concepts will help focus teaching interventions on critical conceptual activities and will help focus neuroscience research on tracking the structural and functional changes that follow intervention. Learning more about how to help these learners is driving, and will continue to drive, where the science should go next.

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ERRATUM

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Review: "Dyscalculia: From brain to education" by B. Butterworth *et al.* (27 May, p. 1049). In Figure 2, A, B, and C, the references should be (*49*), (*50*) and (*51*), respectively.