Prof. Marco Zorzi
University of Padova

Numerical cognition and dyscalculia (part 2)

Numerical development in children

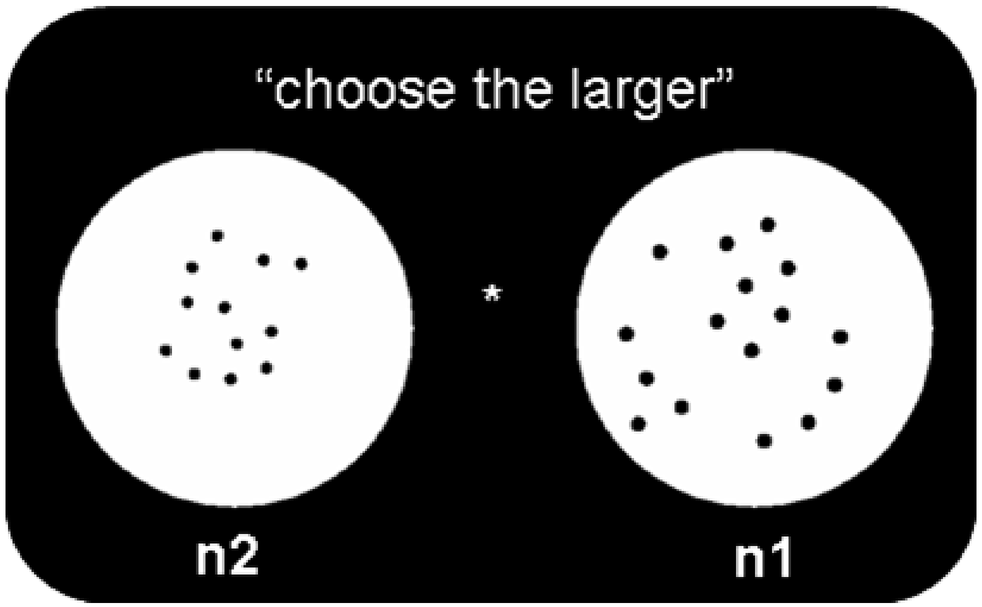
Between 1 and 6 years of age children learn:

- number words
- counting
  - one-to-one correspondence
  - irrelevance of order
  - cardinality principle
- abstract mathematical principles

Table 2. Typical development of whole number competencies in arithmetic [29]

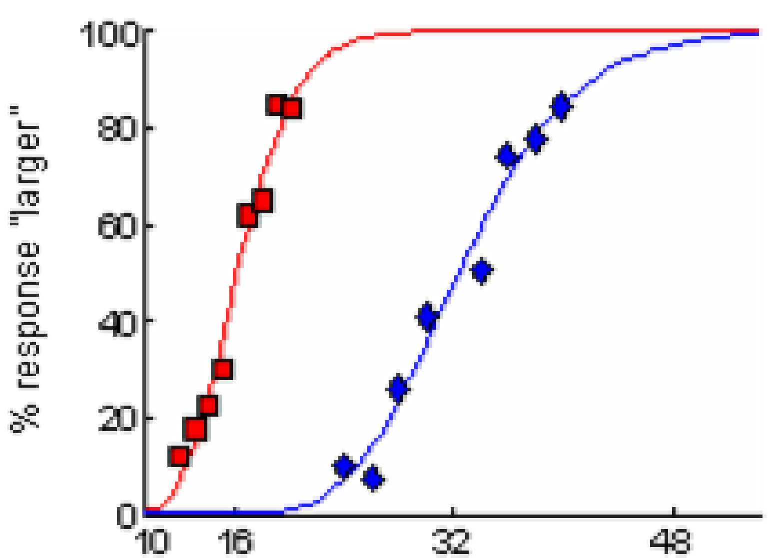
Age	Typical development
0;0	Can discriminate on the basis of small numerosities
0;4	Can add and subtract one
0;11	Discriminates increasing from decreasing sequences of numerosities
2;0	Begins to learn sequence of counting words Can assign one-to-one correspondence in a sharing task
2;6	Recognizes that number words mean more than one ('grabber')
3;0	Counts out small numbers of objects Can recognize transformations that affect number
3;6	Can use the cardinality principle to establish numerosity of set
4;0	Can use fingers to aid adding
5;0	Can add small numbers without being able to count out sum
5;6	Understands commutativity of addition
6;0	Piagetian 'conservation of number'
6;6	Understands complementarity of addition and subtraction
7;0	Retrieves some arithmetic facts from memory

Measuring “number acuity”: the Weber fraction (w)

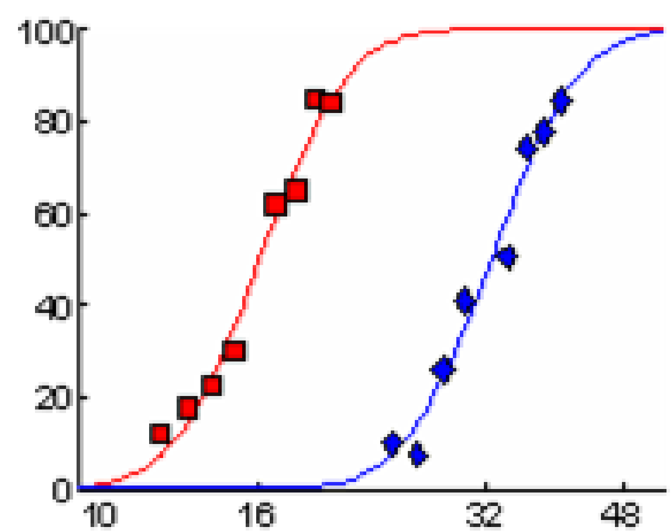


Numerosity comparison
n1: reference number (16 or 32);
n2: variable number

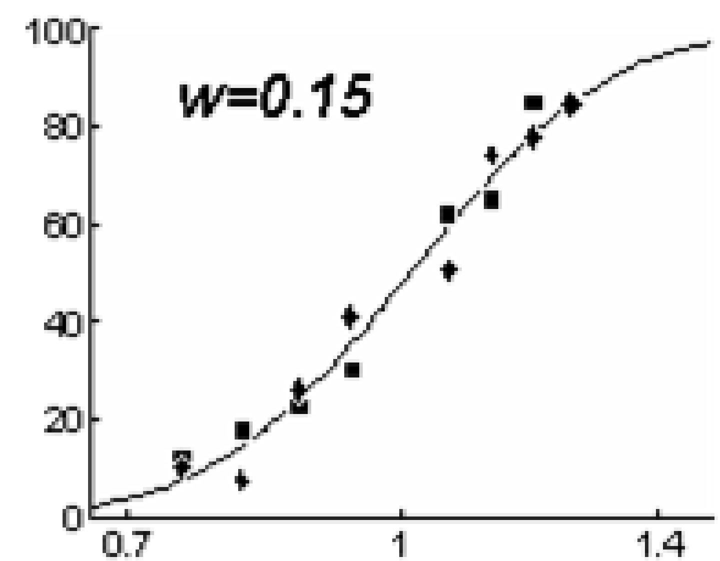
w is the standard deviation of the internal representation (Gaussian distribution) that best fits the behavioral data. It is an index of discriminability between two numerosities.



n2 numerosity (linear scale

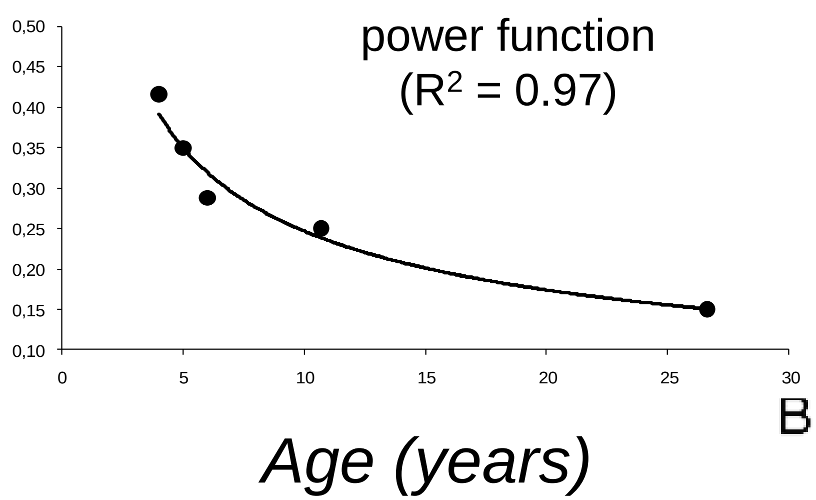


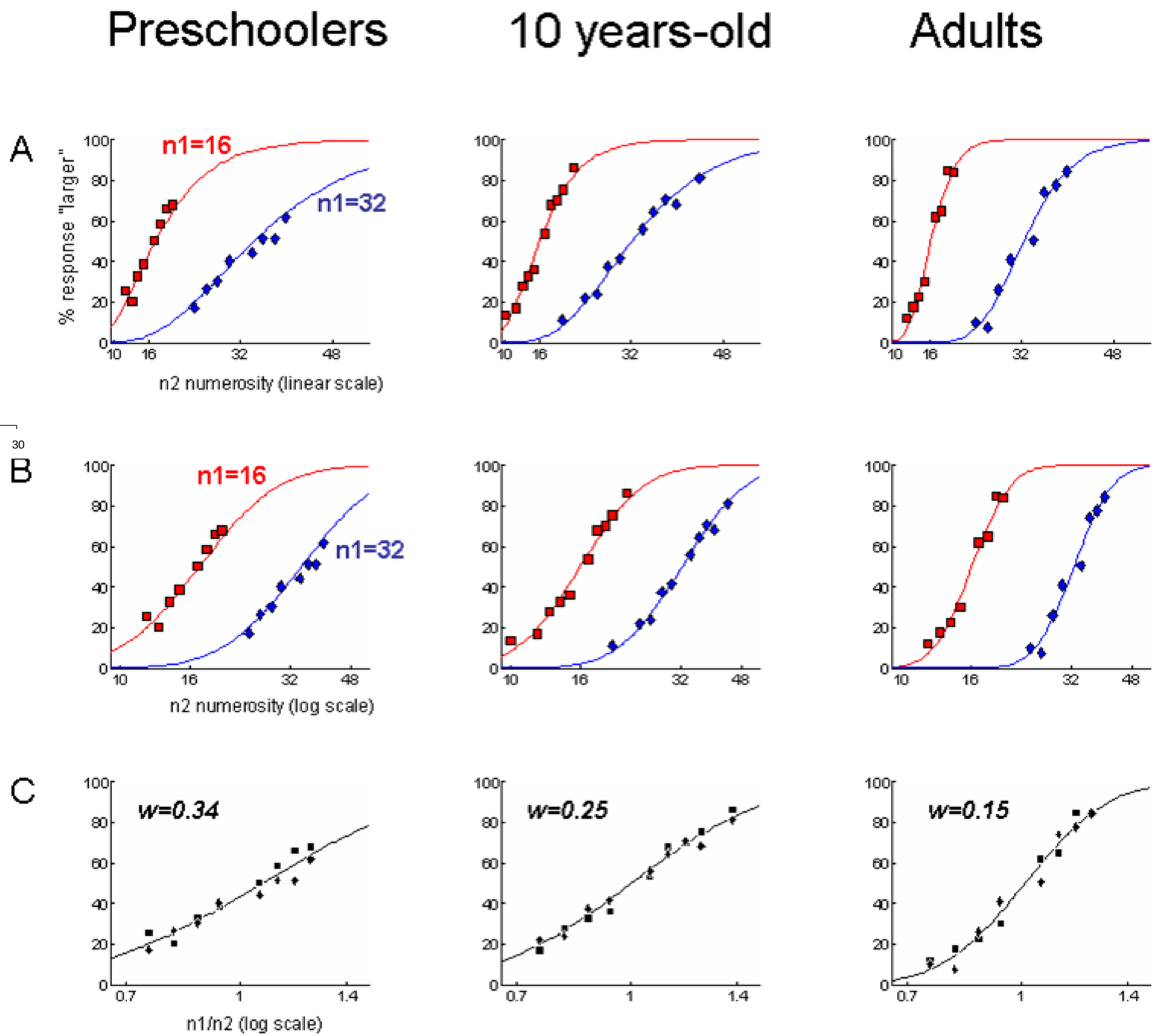
n2 numerosity (log scale



n1/n2 (log scale)

Typical development of *number acuity*

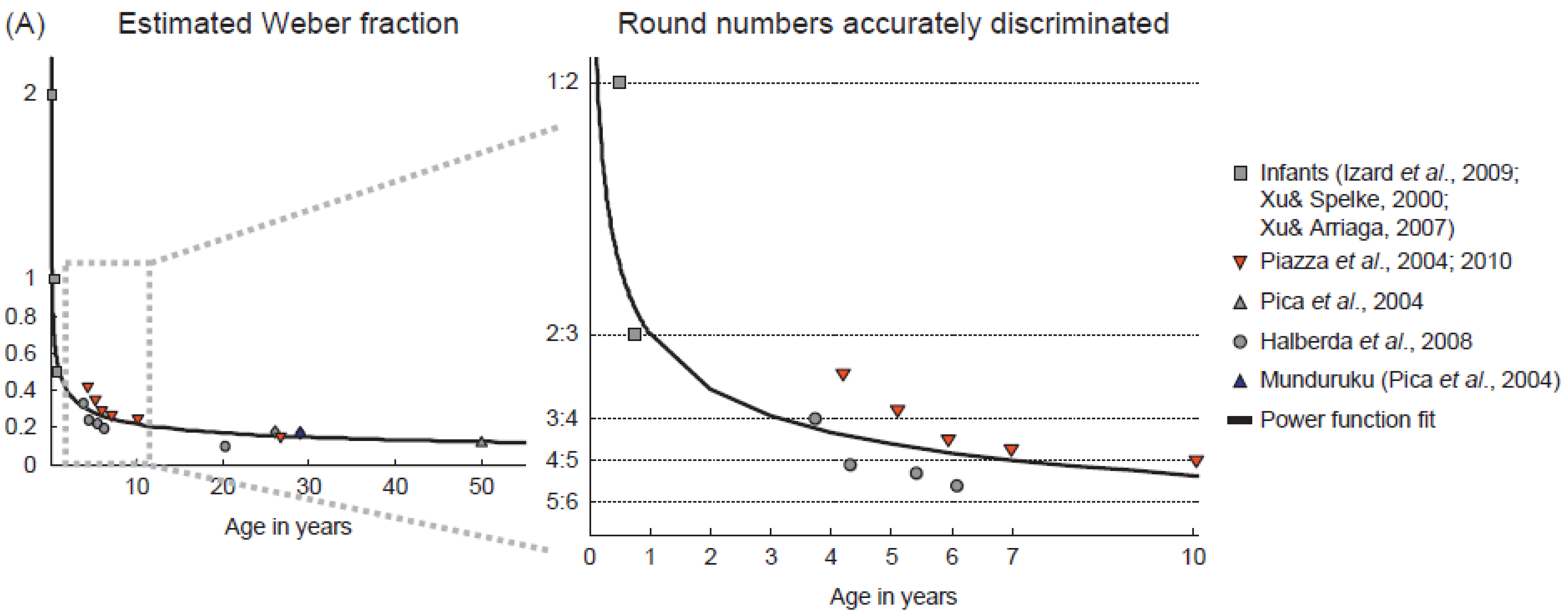




Number acuity improves throughout childhood

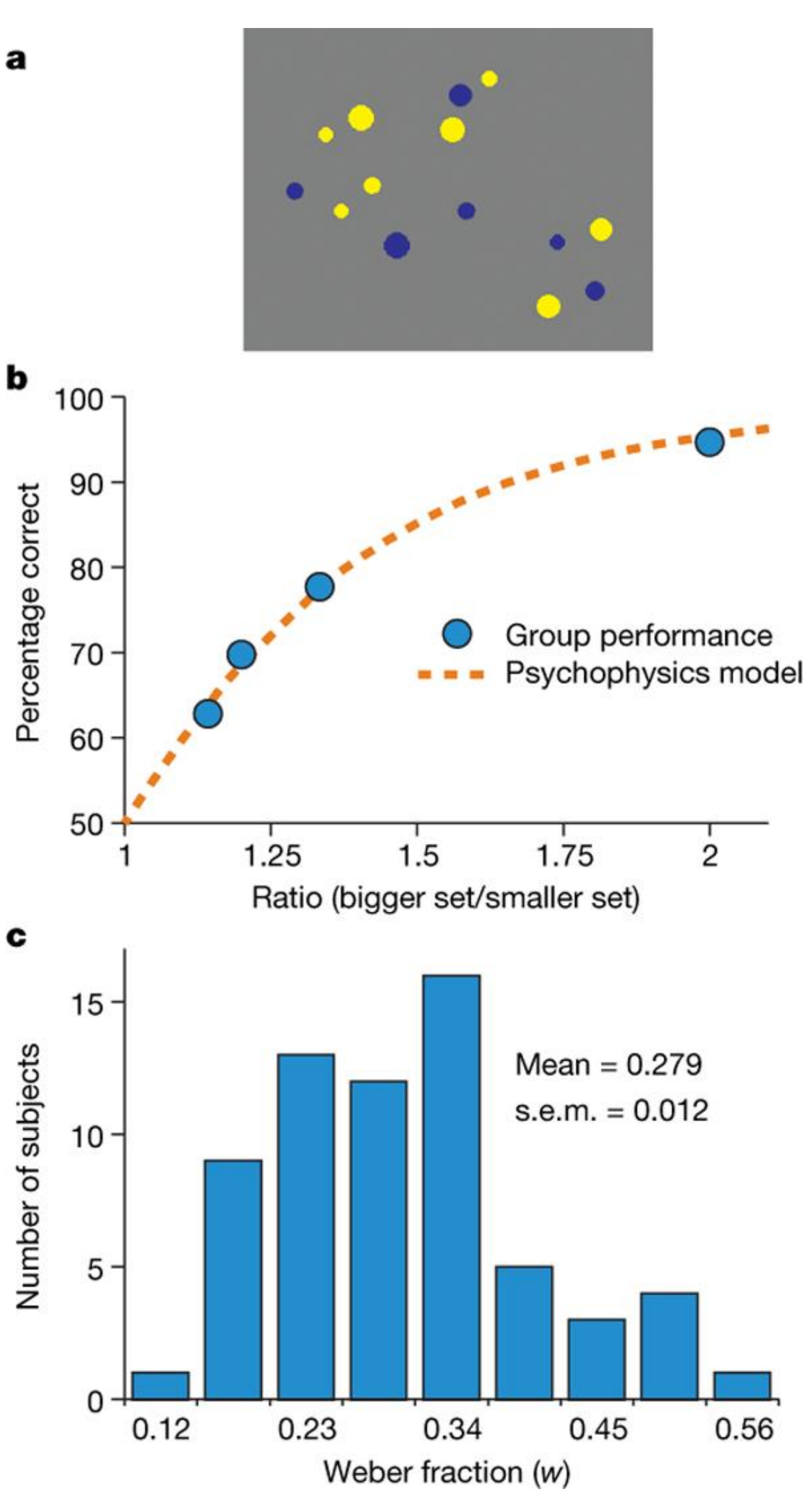
Piazza,.., Zorzi, 2010, *Cognition*

Typical development of number acuity

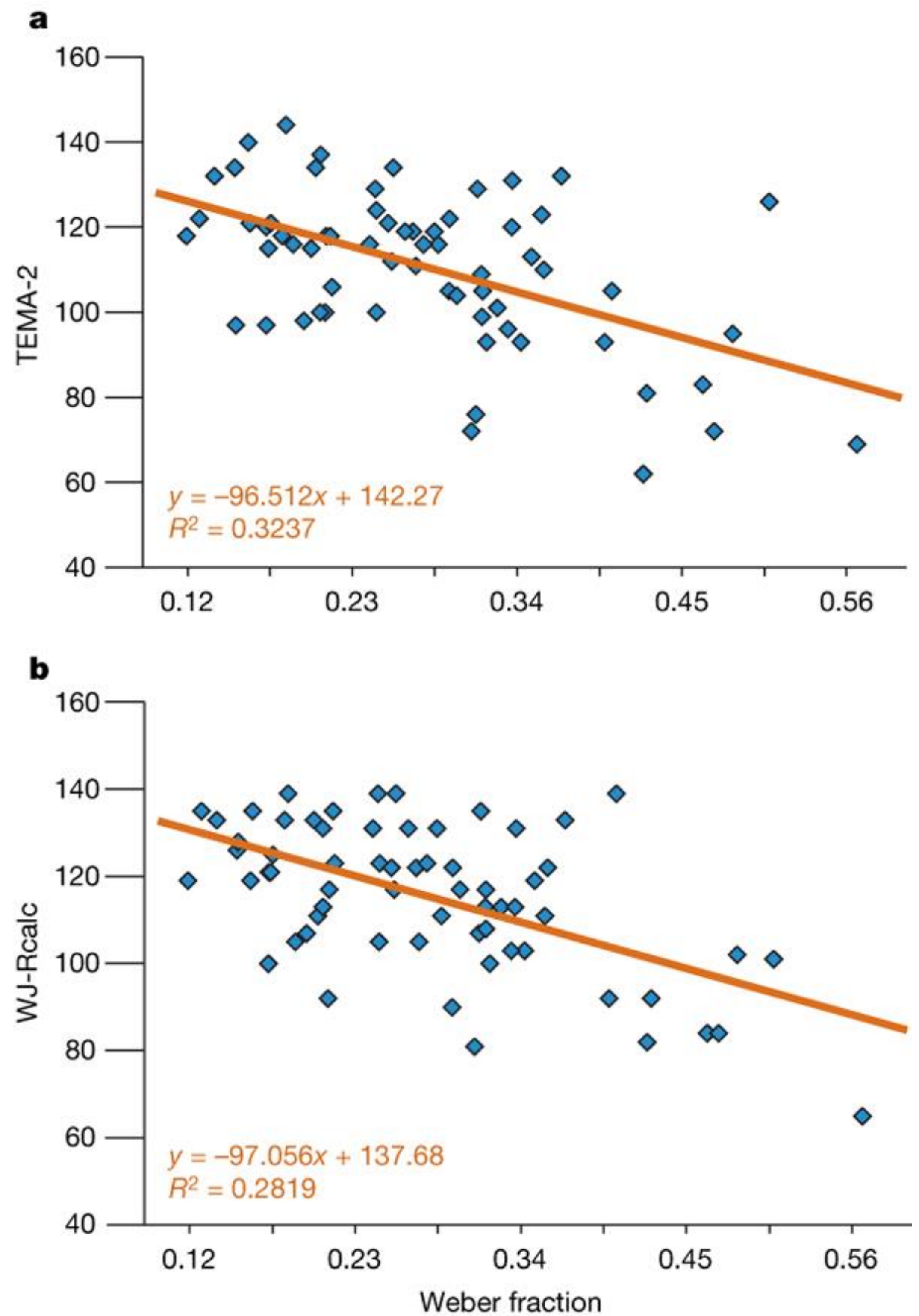


Piazza, 2011, *TICS*

Number acuity predicts math achievement

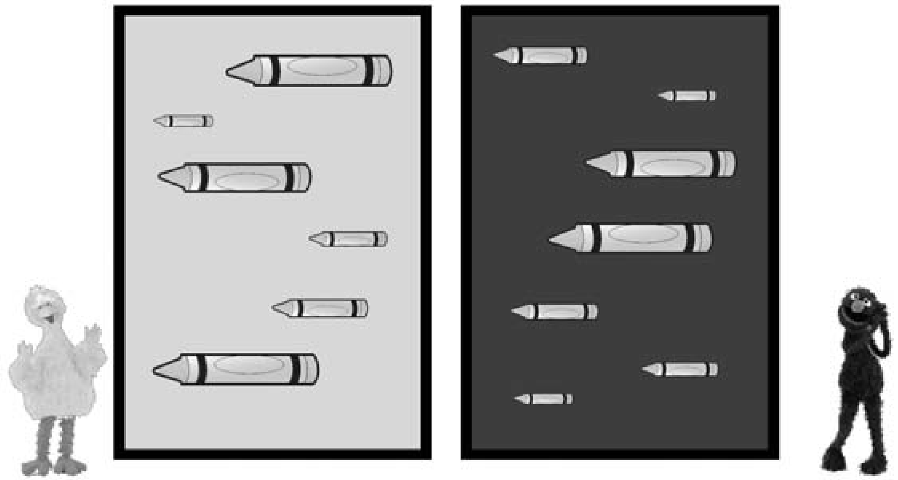


64 children 14-15 y.o.



Correlations with math achievement tests at 8 y.o.

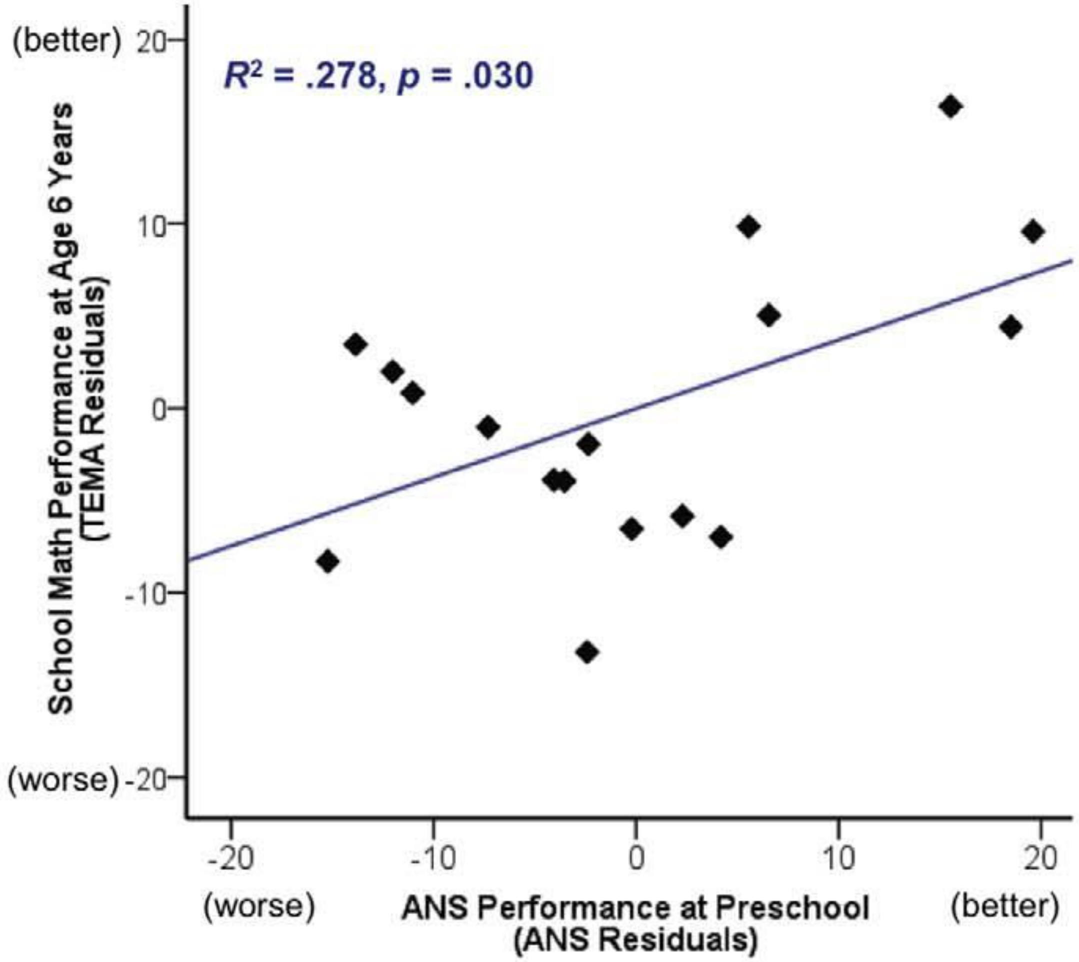
Halberda et al., 2008, *Nature*

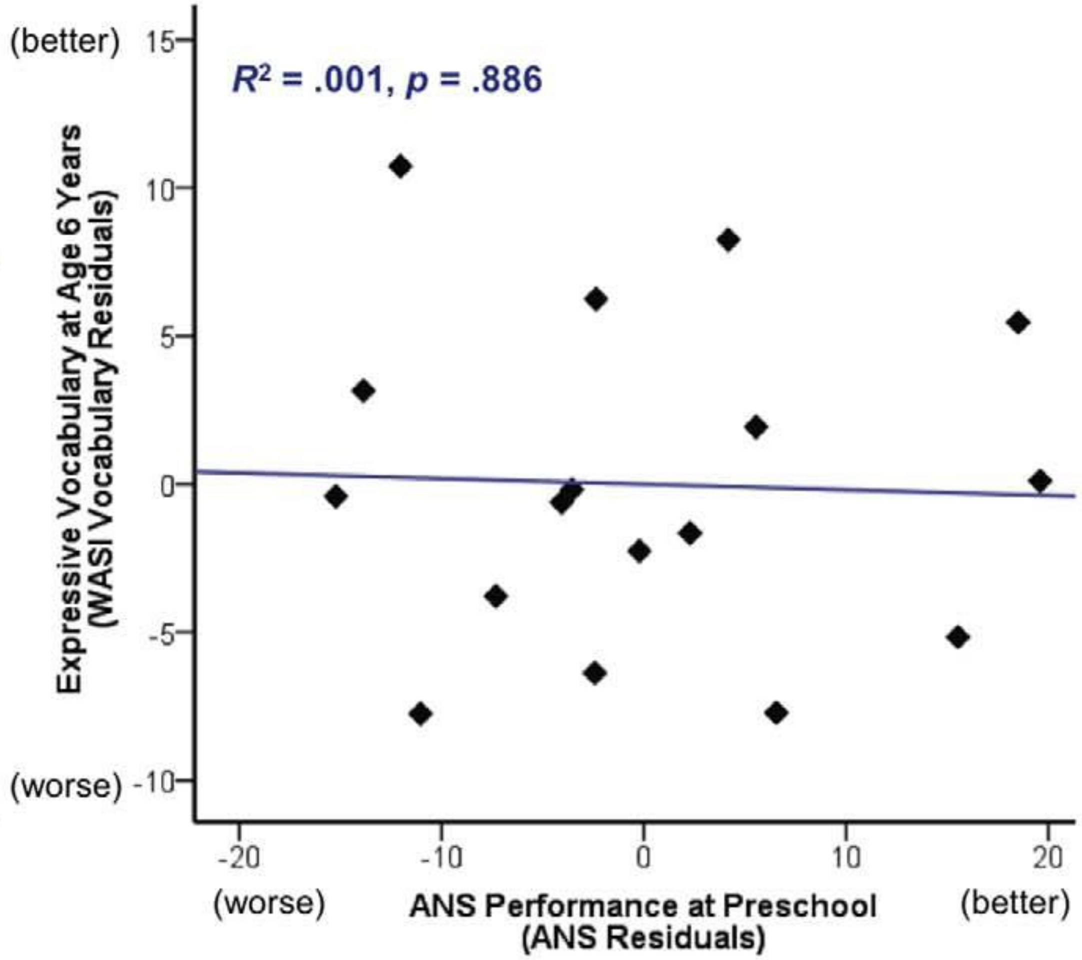


"Who has more crayons?"

Table 1. Participant ages at, and time intervals between, preschool and follow up assessments in years and months (N = 17).

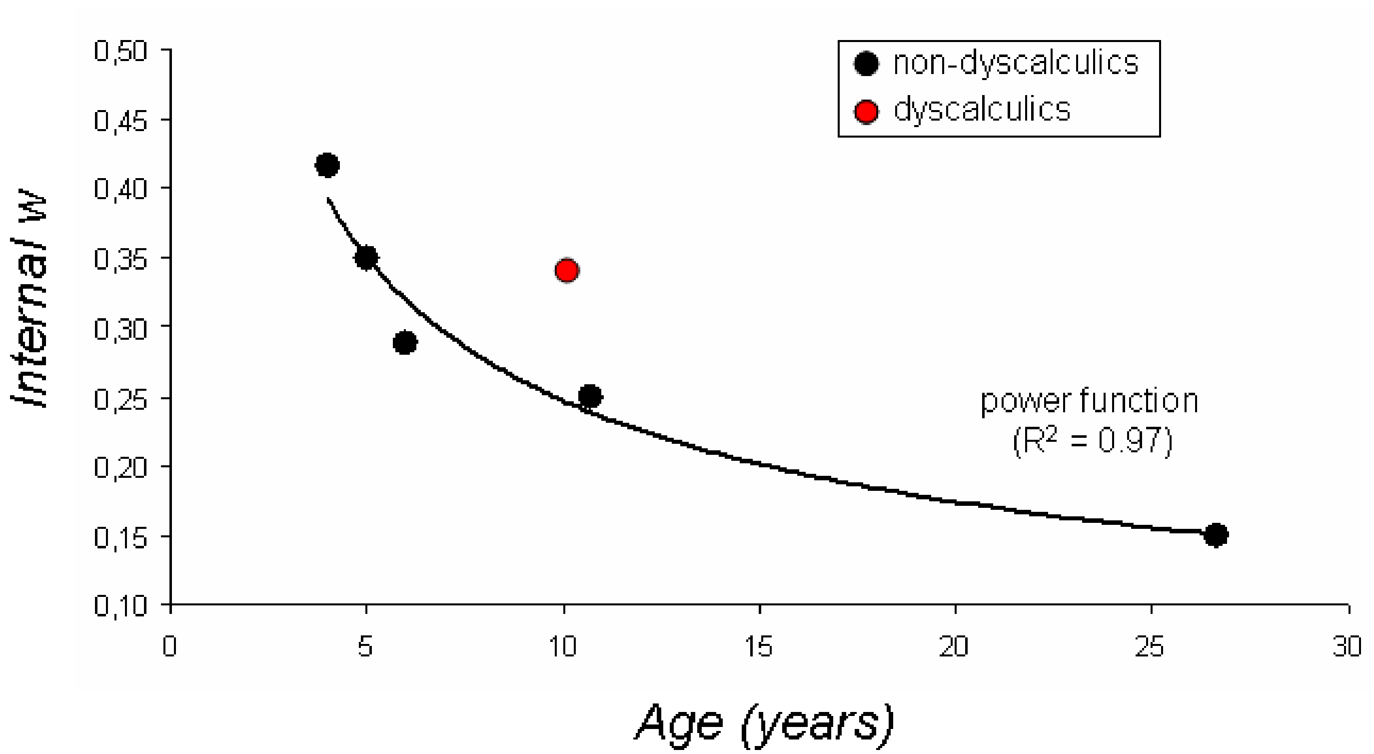
Time Point	Mean	Std. Deviation	Age Range
Age at Preschool	4; 2	0; 4.5	3; 5 to 4; 11
Age at Follow up	6; 8	0; 4.2	6; 2 to 7; 5
Interval between assessments	2; 6	0; 2.8	2; 0 to 2; 9

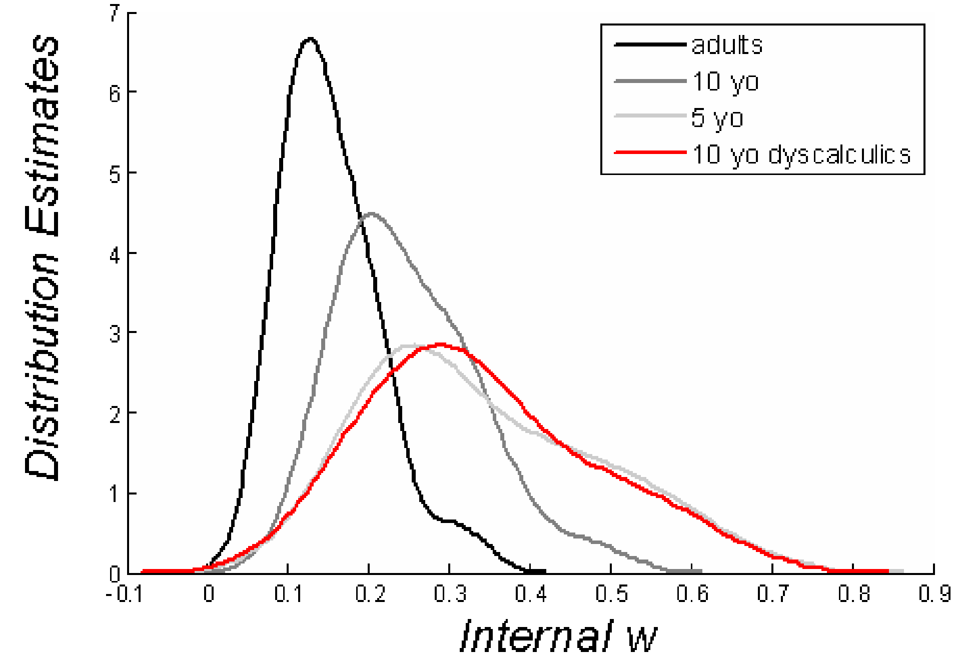


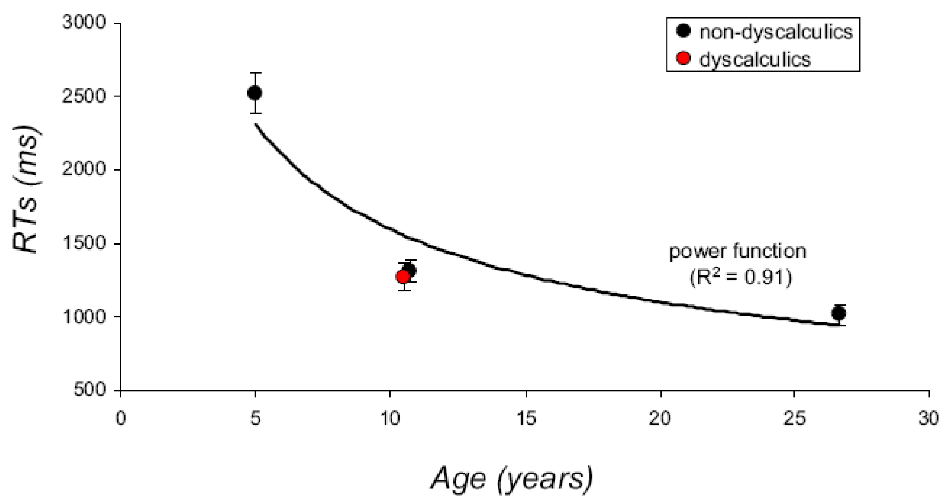


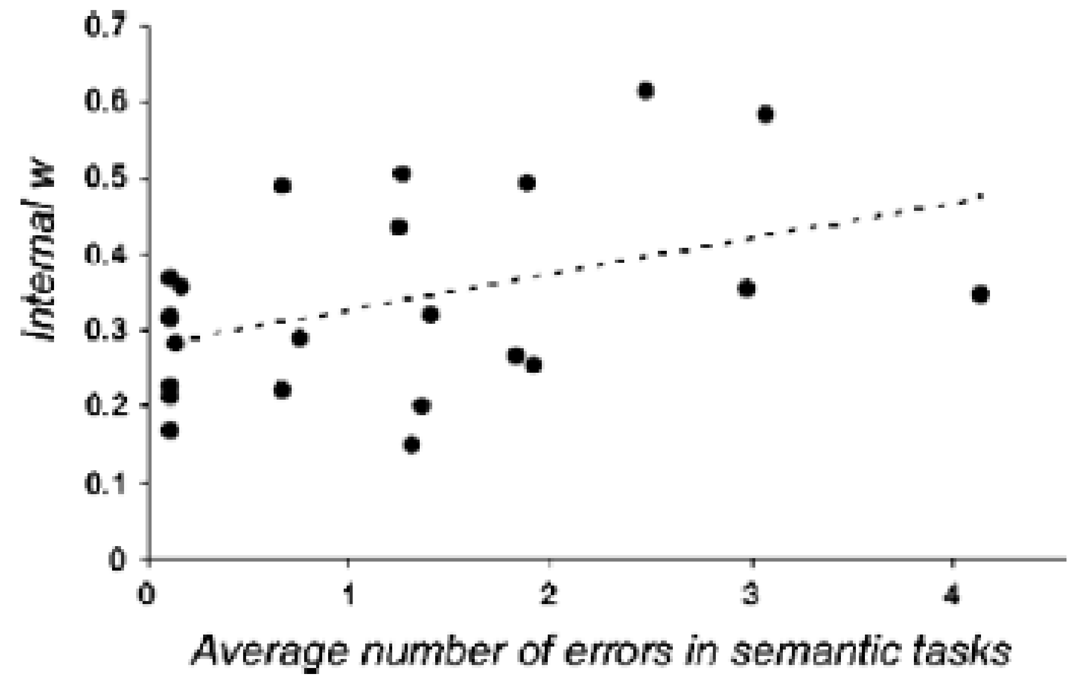
Mazzocco et al., 2011, Plos One

Number acuity (ANS) is impaired in developmental dyscalculia

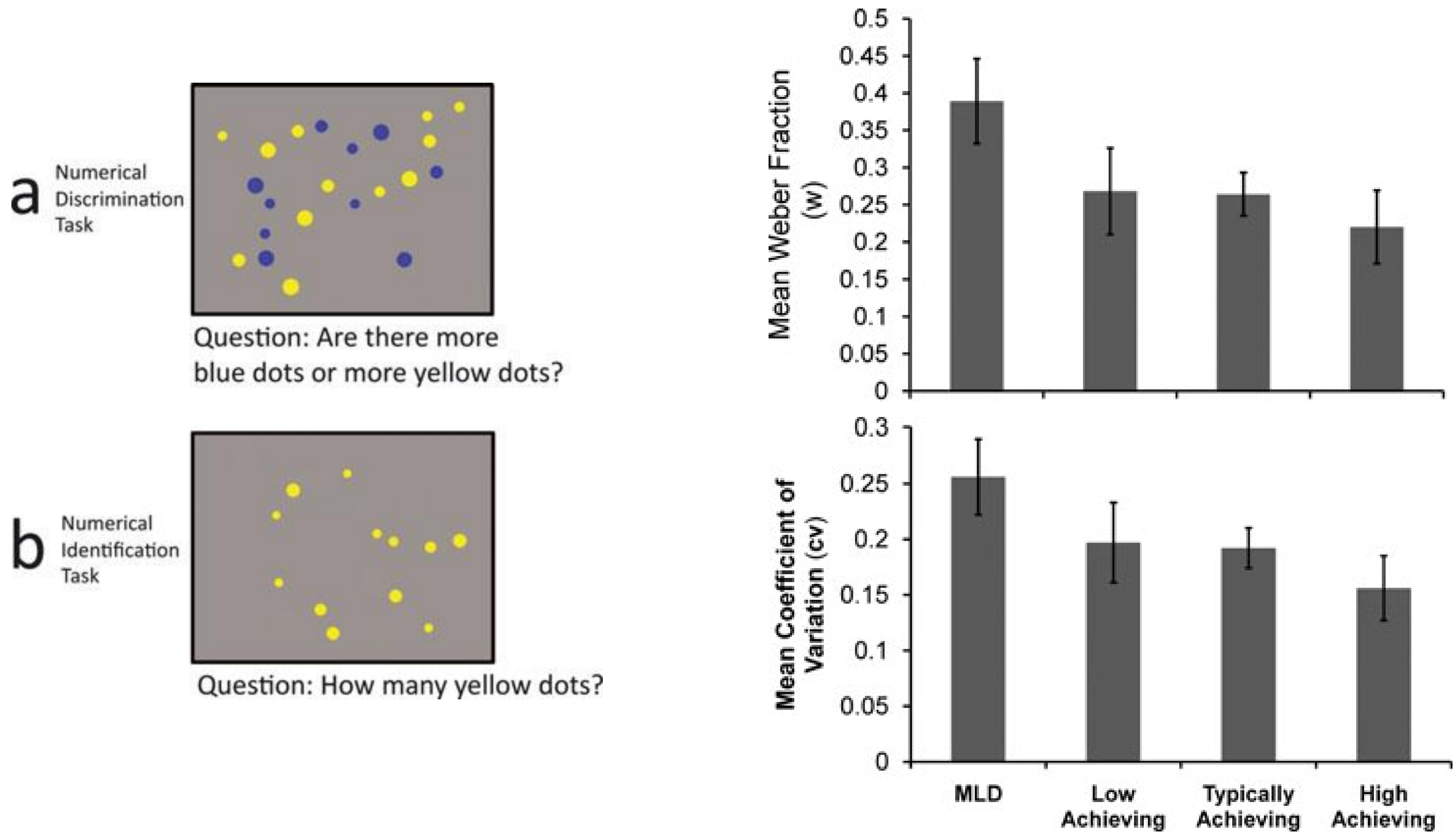






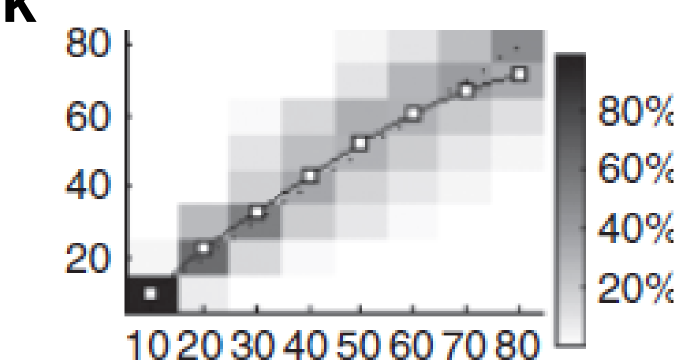


Piazza,.., Zorzi, 2010, *Cognition*



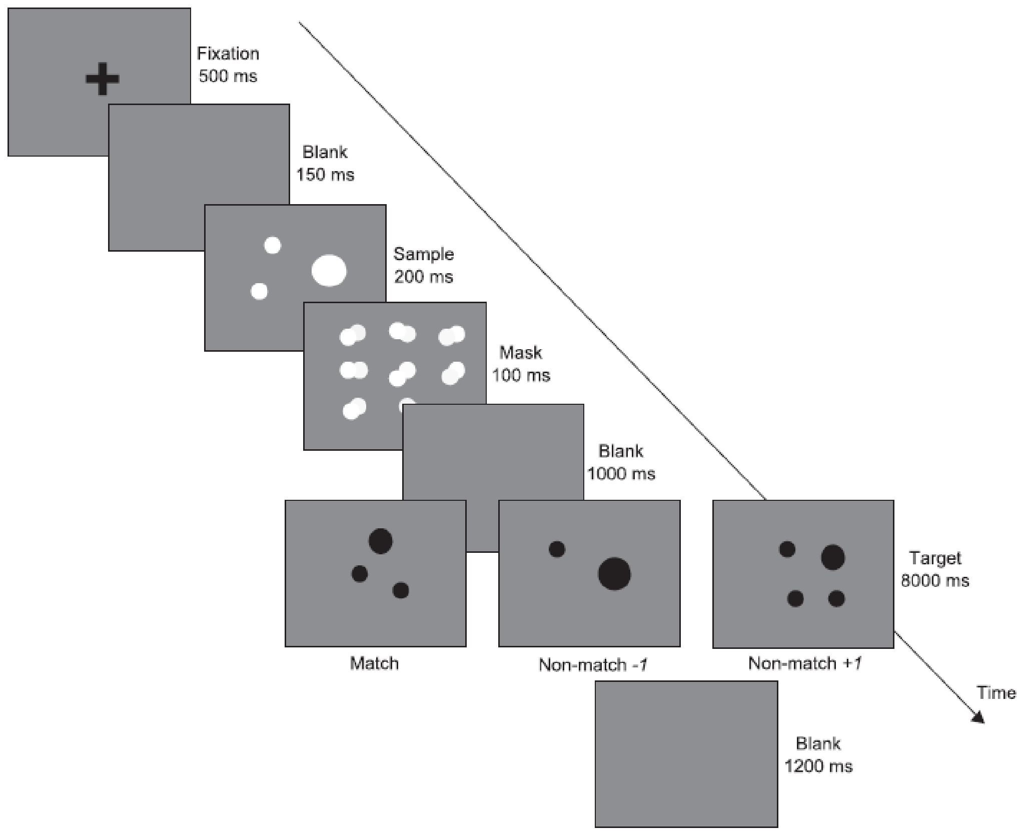
Coefficient of variation (CV) is an index of precision in the **numerosity naming task**

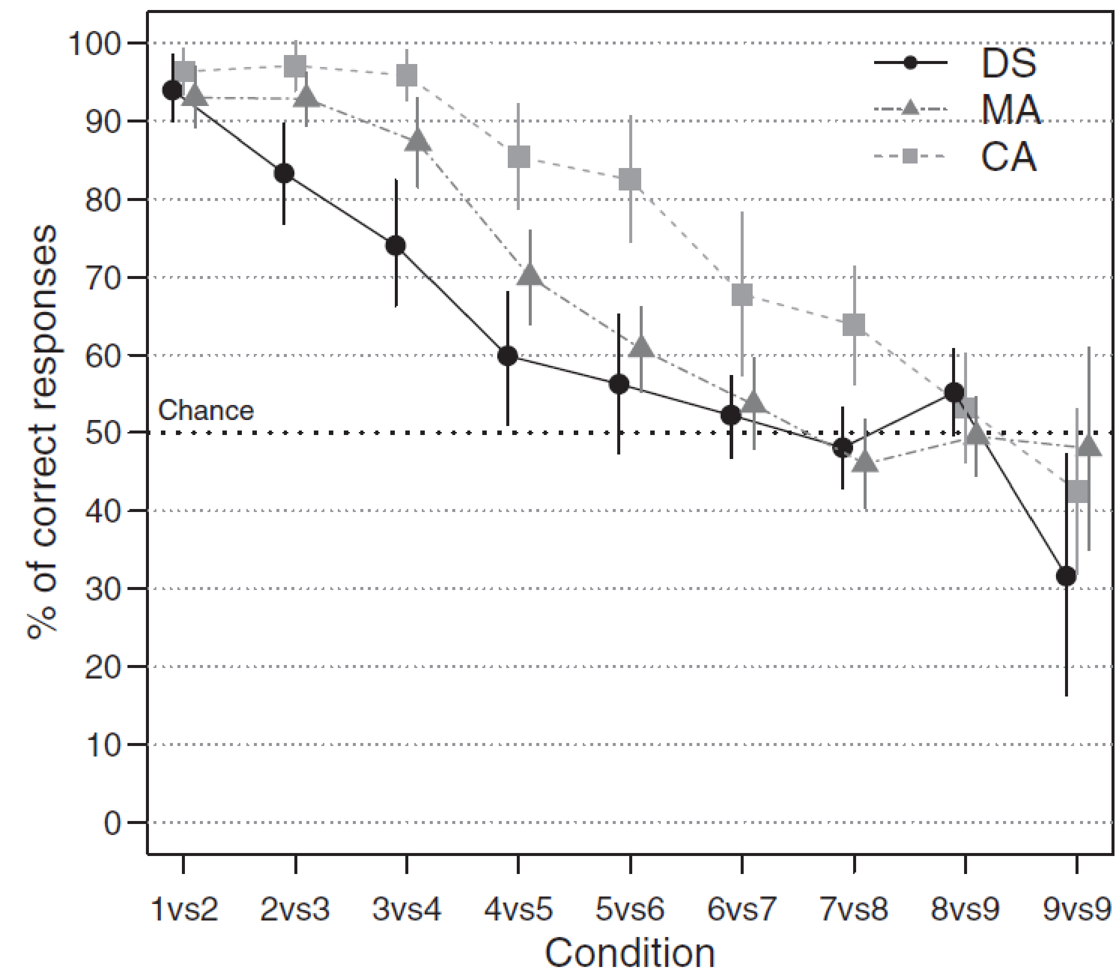
CV: standard deviation of response divided by mean response

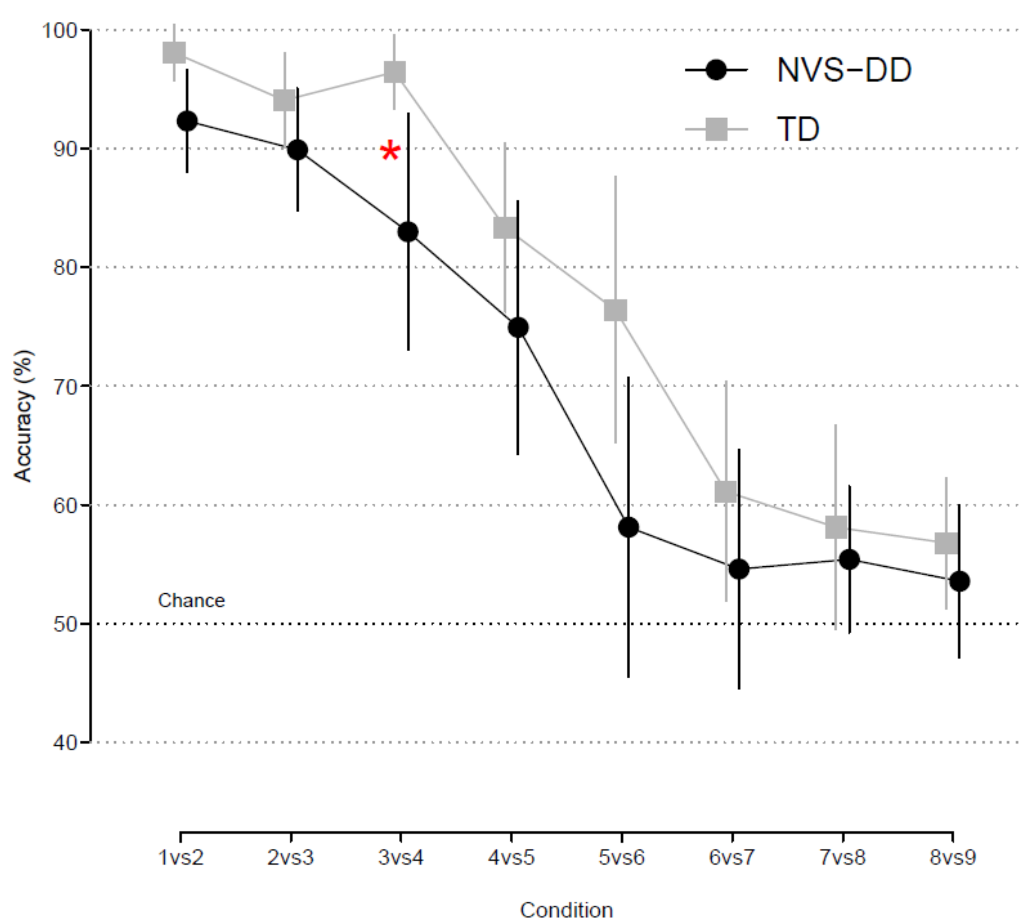


Mazzocco et al., 2011, Child Development

Deficit in subitizing (OTS) in DS and DD



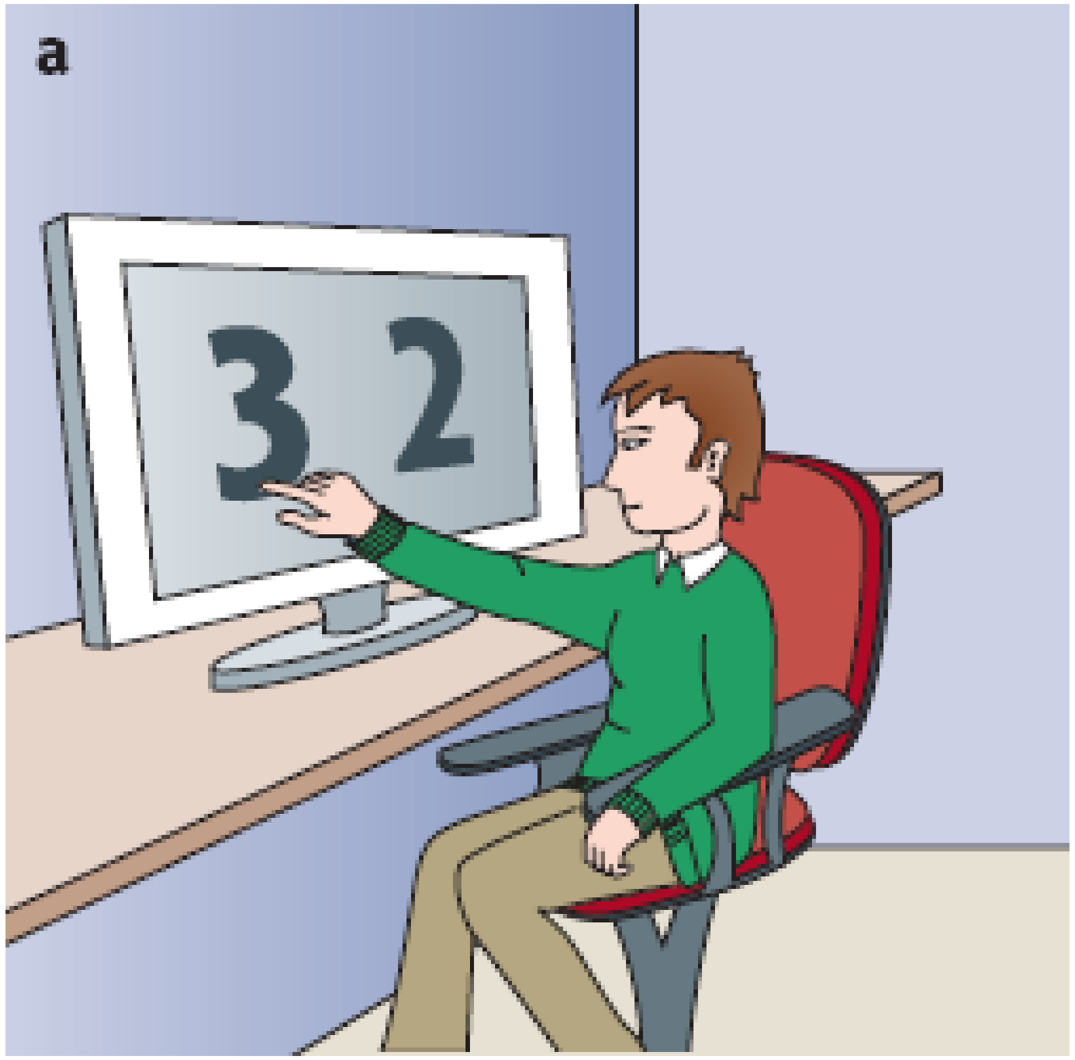




A subitizing deficit is found in children with Down Syndrome (DS), who are known to have math learning difficulties, even in comparison to controls matched for mental age (MA). (Sella, Lanfranchi, & Zorzi, 2013, *Res. Dev. Disabil.*)

A similar deficit is found in children with developmental dyscalculia (DD) in comparison to typically developing (TD) children. (Sella et al., in preparation)

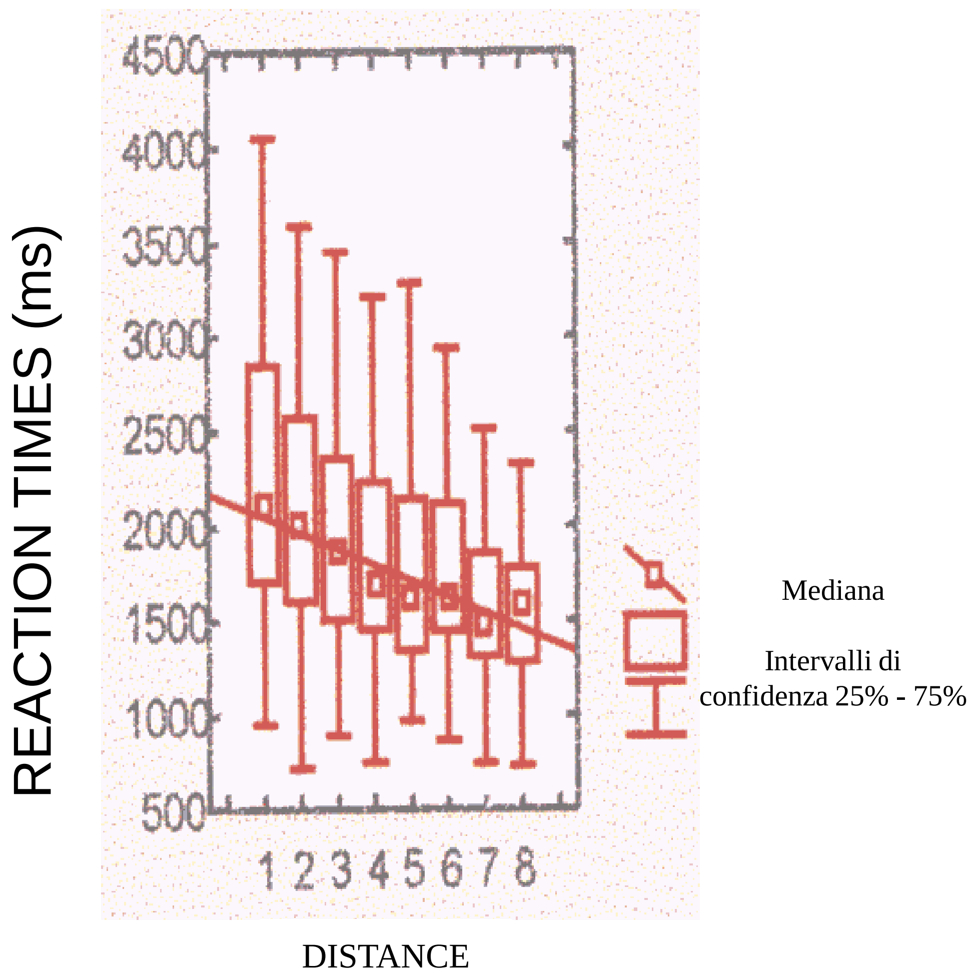
Beyond the “number sense”



Distance effect in symbolic number comparison

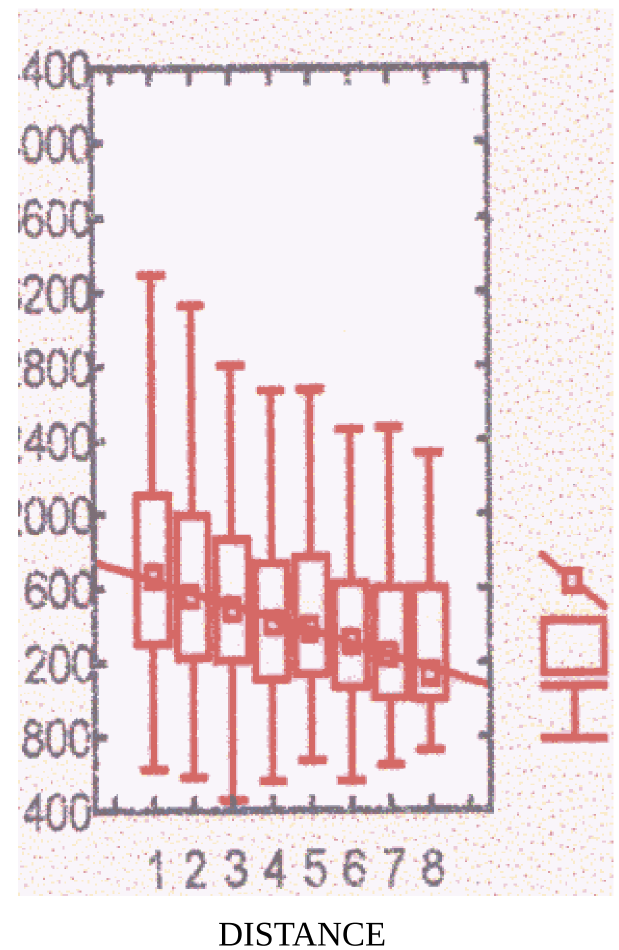
5 years

RT = 2279,62 –87,83 * distance



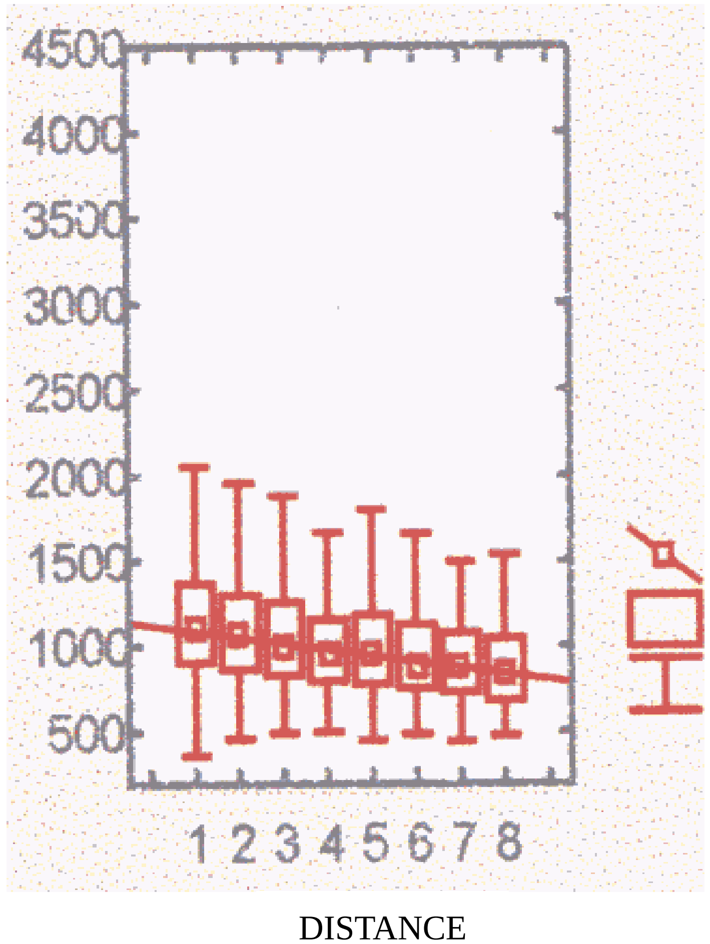
6 years

RT = 1774,34 – 58,47 * distance



8 years

RT = 1211,78 – 37,34 * distance

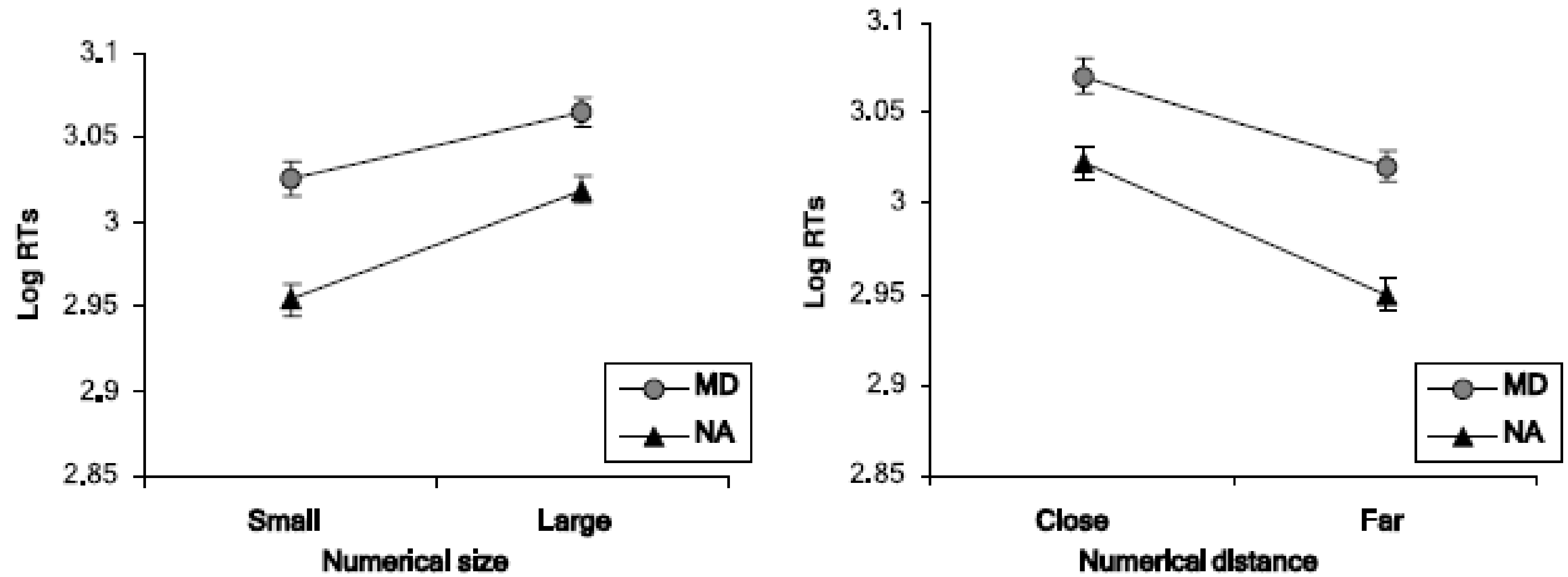


The slope of the distance effect decreases as a function of age. Increasing representational precision?

(Lucangeli, Zorzi, Cabele, 2006)

Number comparison in developmental dyscalculia:

Rouselle e Noel, 2006
Cognition



Dyscalculics are slower and less accurate than controls in Arabic number comparison. Reduction of distance and size effect in DD.

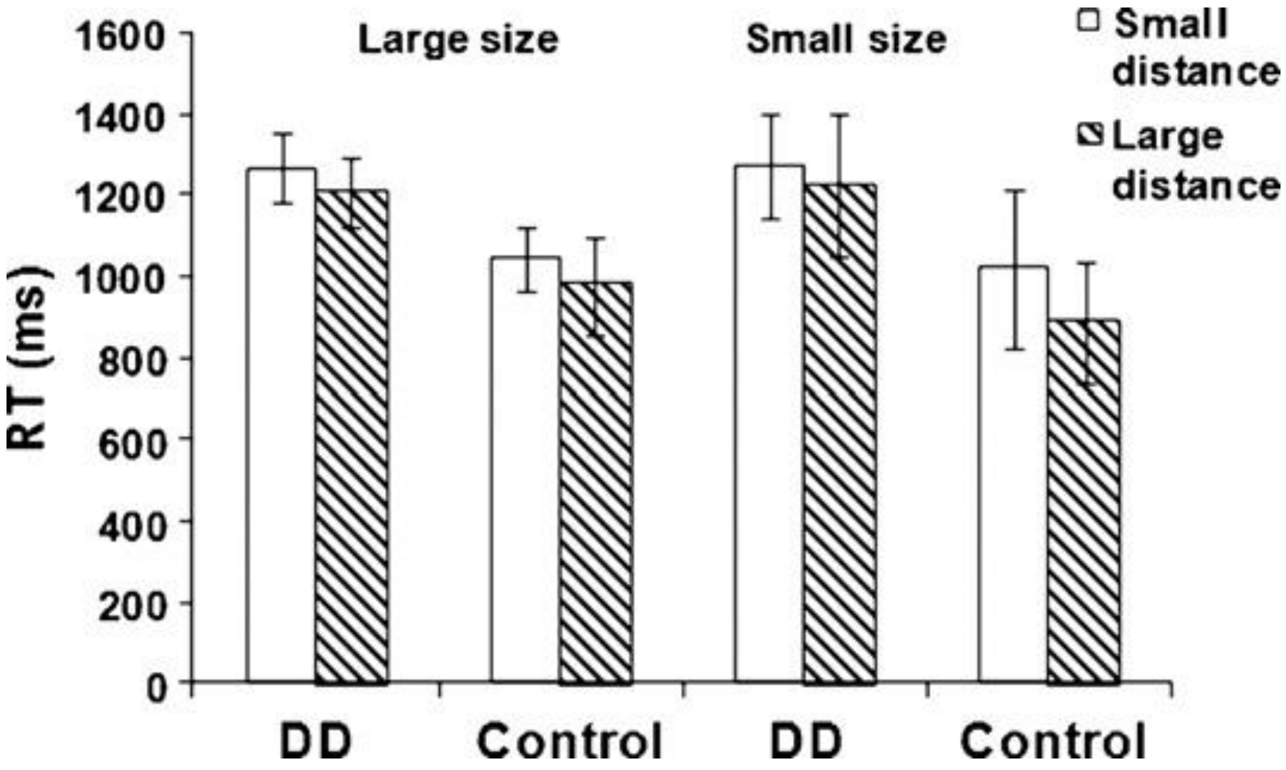
No difference between DD and controls in non-symbolic comparison.

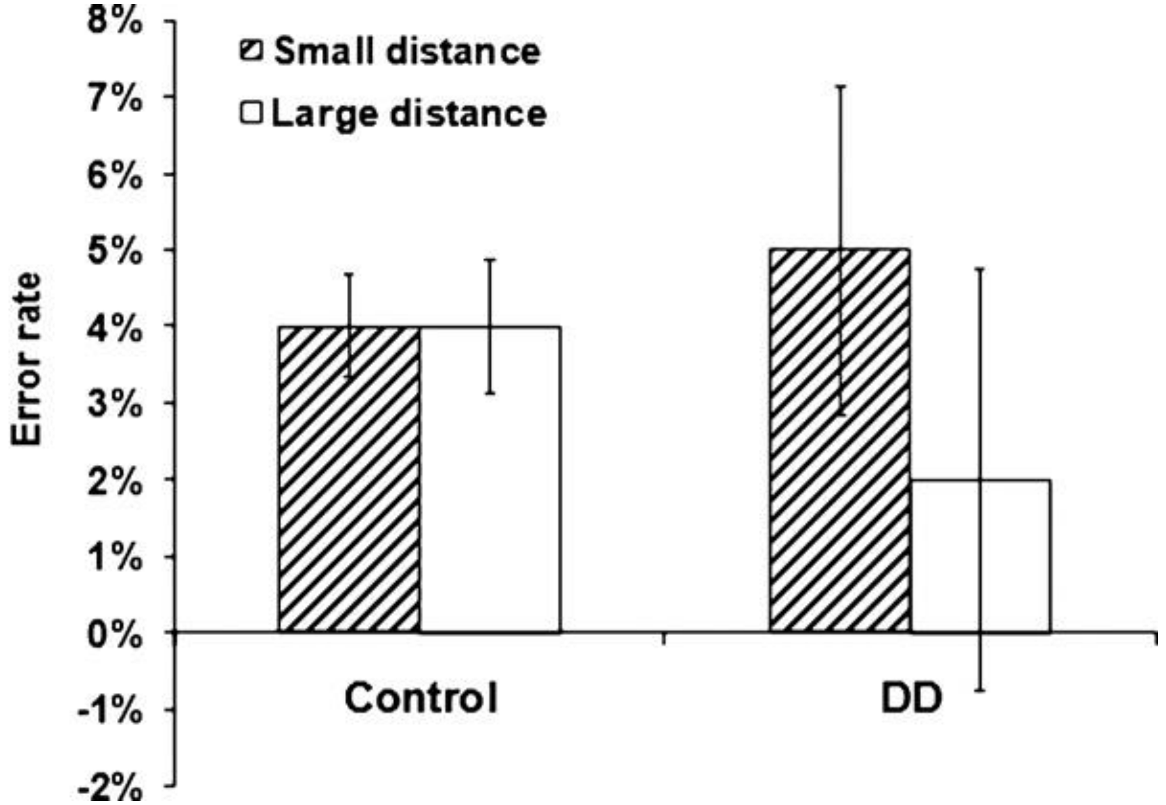
R&N conclude that the main difficulty is to access numerical magnitude from symbols (**access deficit**); the analog representation of numerical magnitude would be intact. Similar conclusion in Iuculano et al. (2008, *Developmental Science*).

Problems: i) sensibility of non-symbolic test, ii) emphasis on RTs, iii) potential subject selection issues

As shown in Piazza et al. (2010), non-symbolic comparison is impaired, which implies a **number sense deficit** in DD

Symbolic distance effect in DD

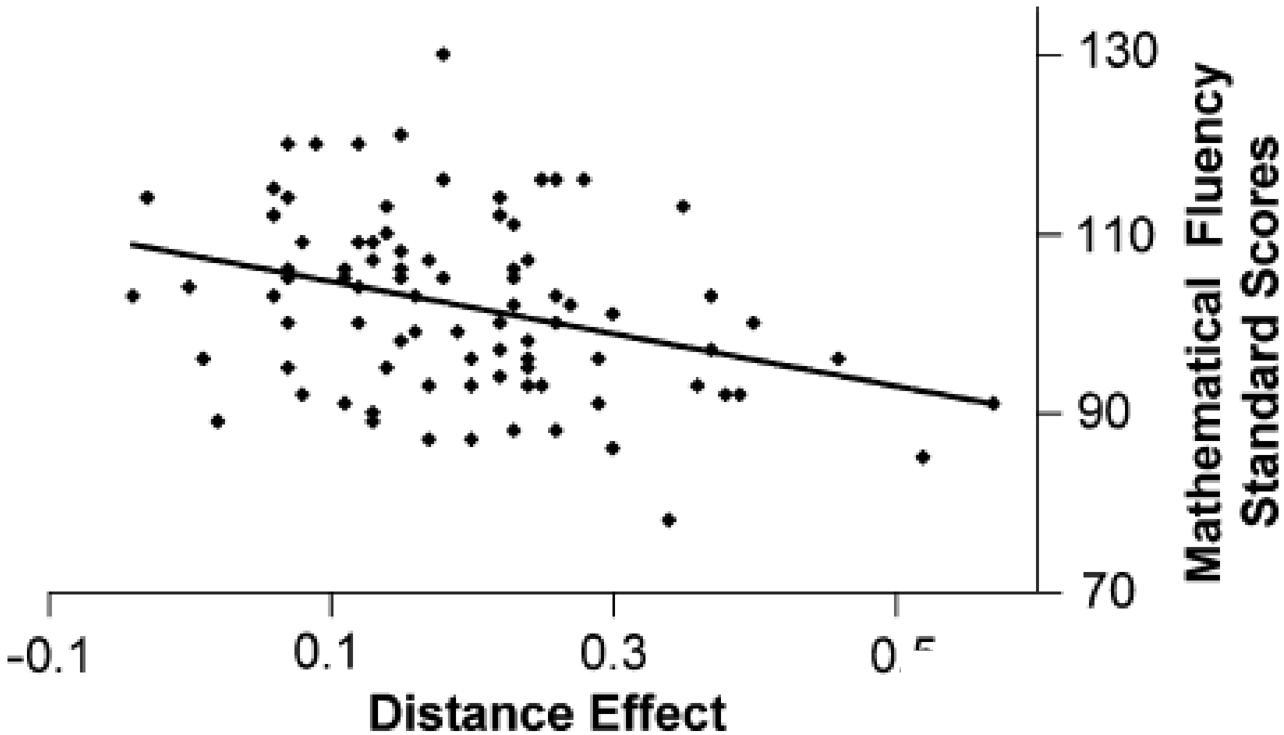


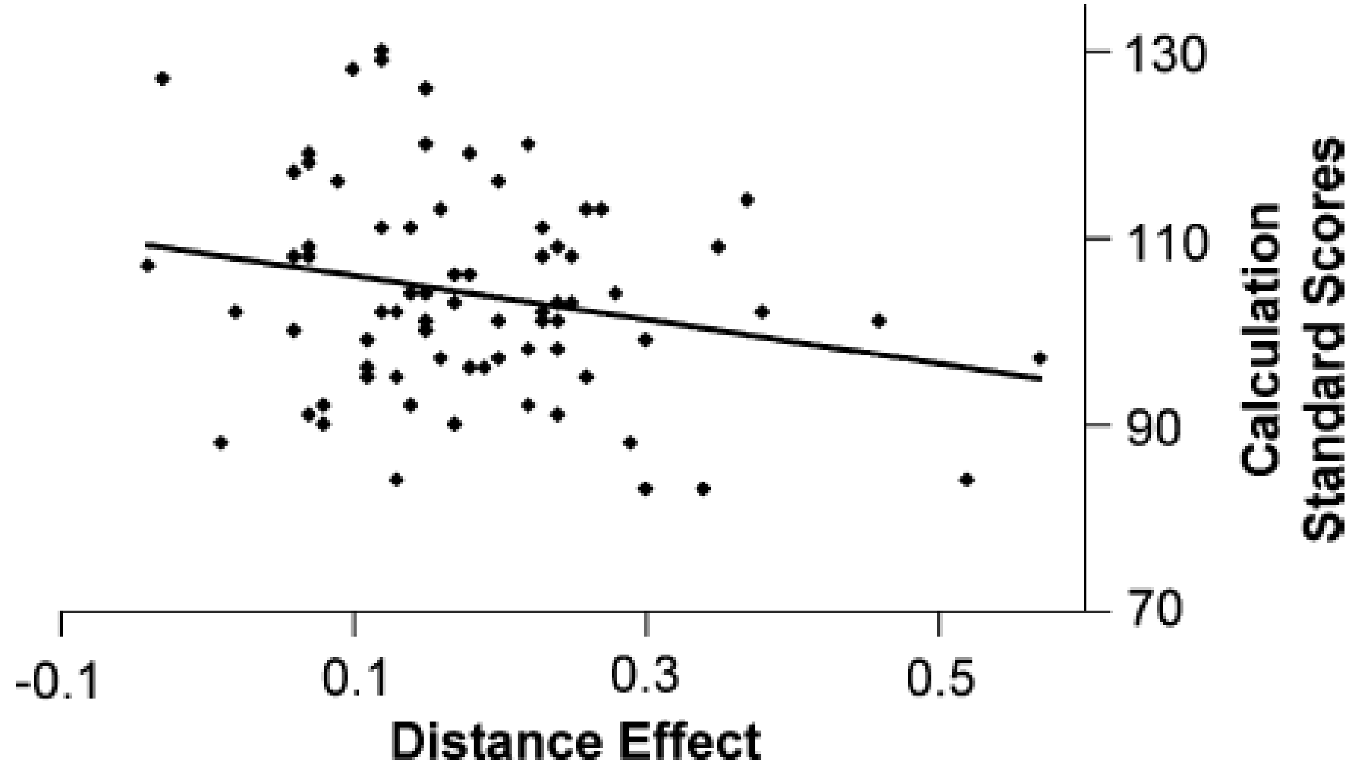


N=13 DD,
16 CA

Ashkenazi et al., 2009, Cog. Dev.

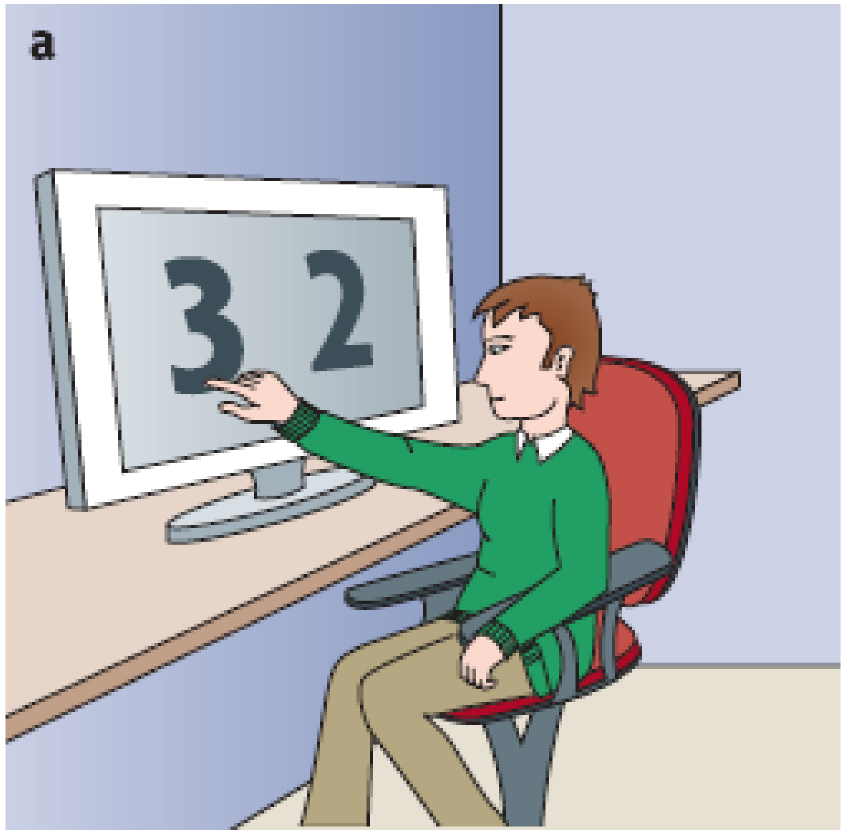
Distance effect and mathematical abilities





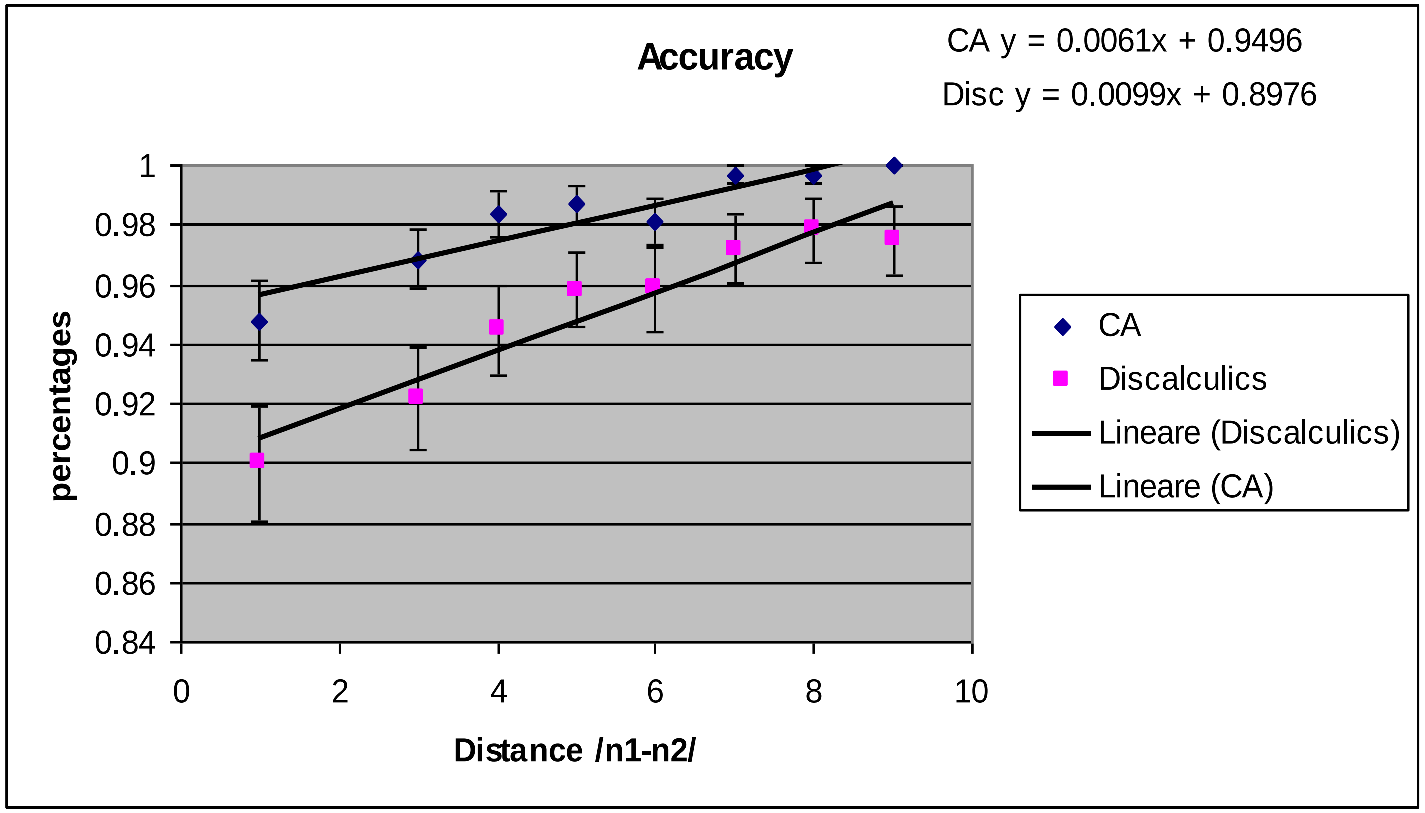
Holloway & Ansari, 2009

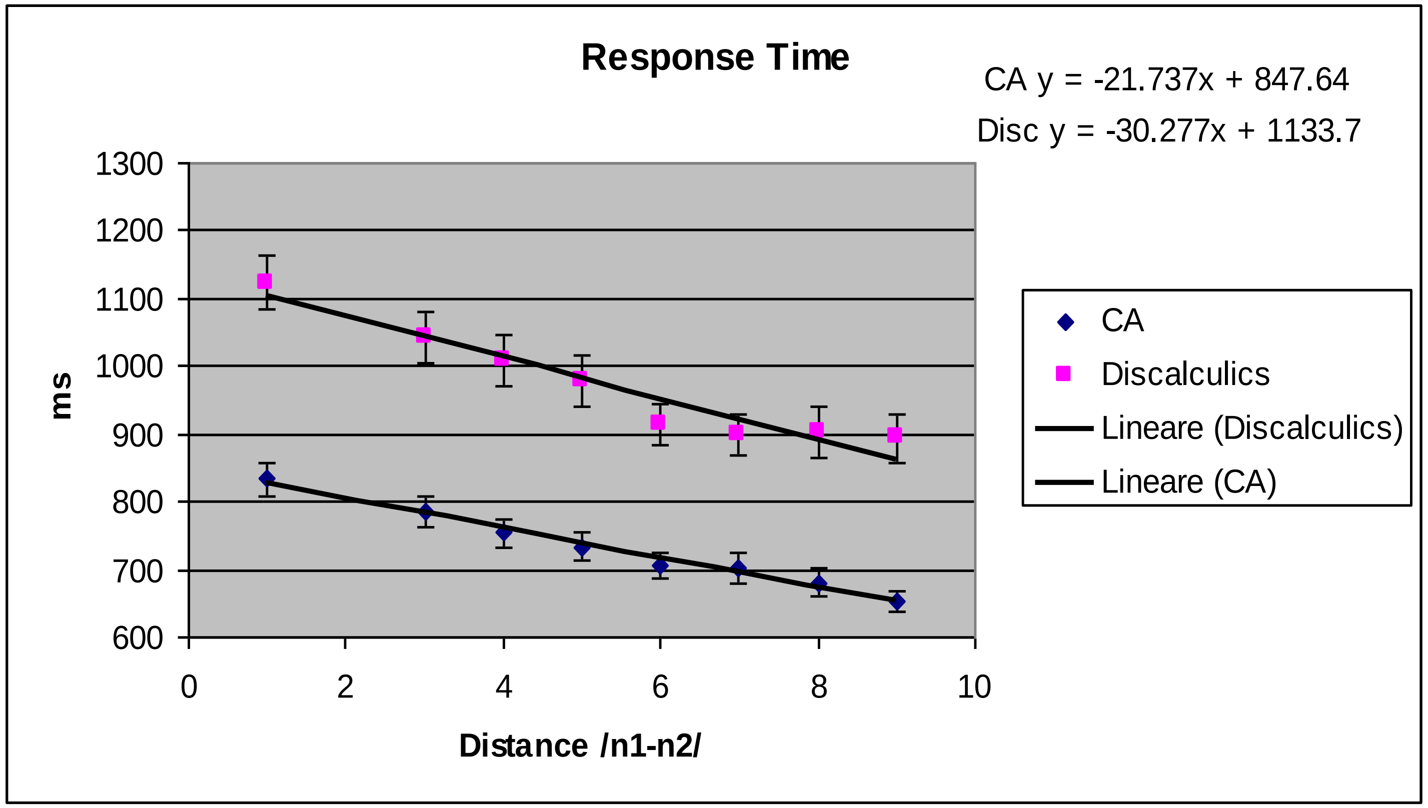
Symbolic number comparison in DD



- N
  - Dyscalculic children (DD) = 69
  - Chronological age controls (CA) = 78
  - Younger controls (YC) = 19
- Age
  - DD = 124.8 (SD = 14)
  - CA = 124.3 (SD = 13.8)
  - YC = 93.6 (SD = 7.5)

Zorzi et al., in preparation



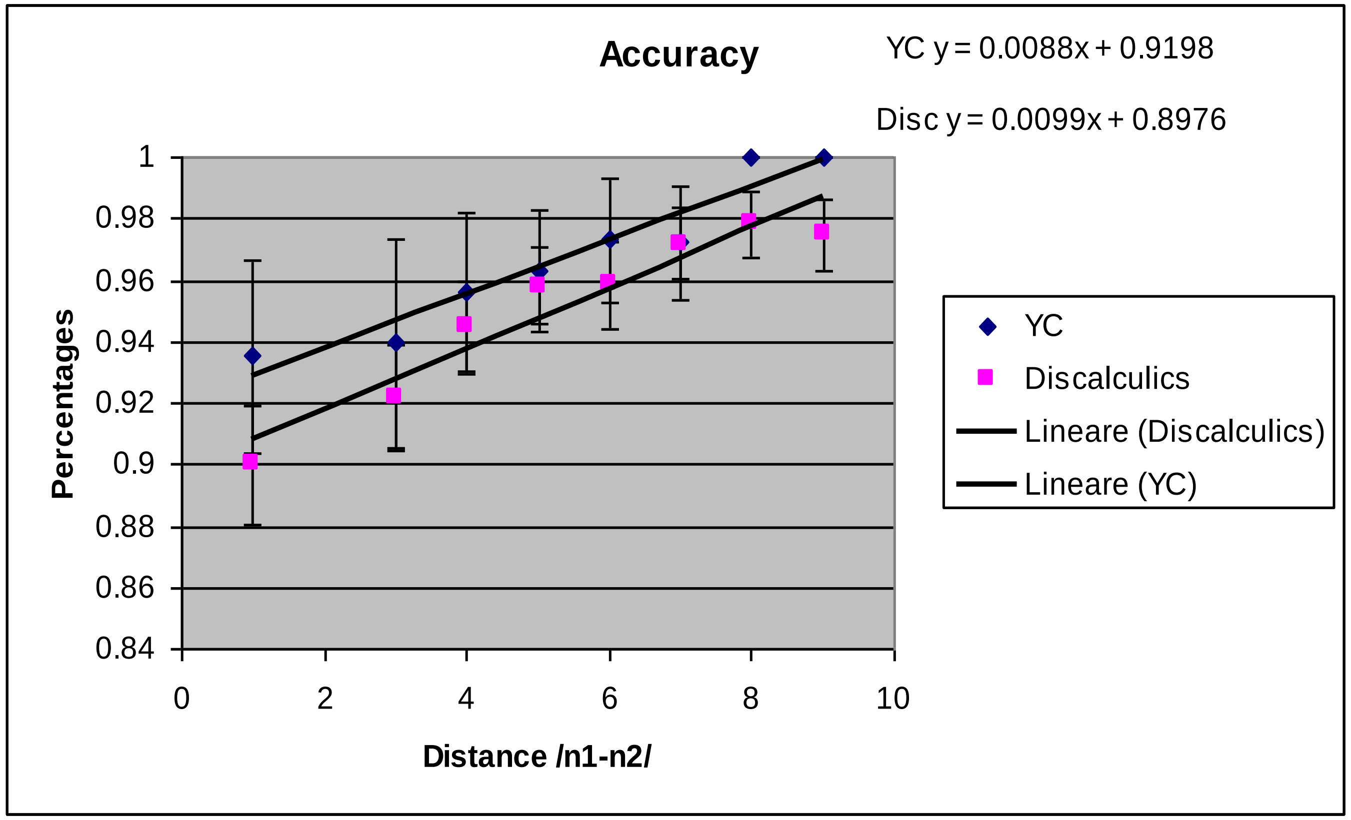


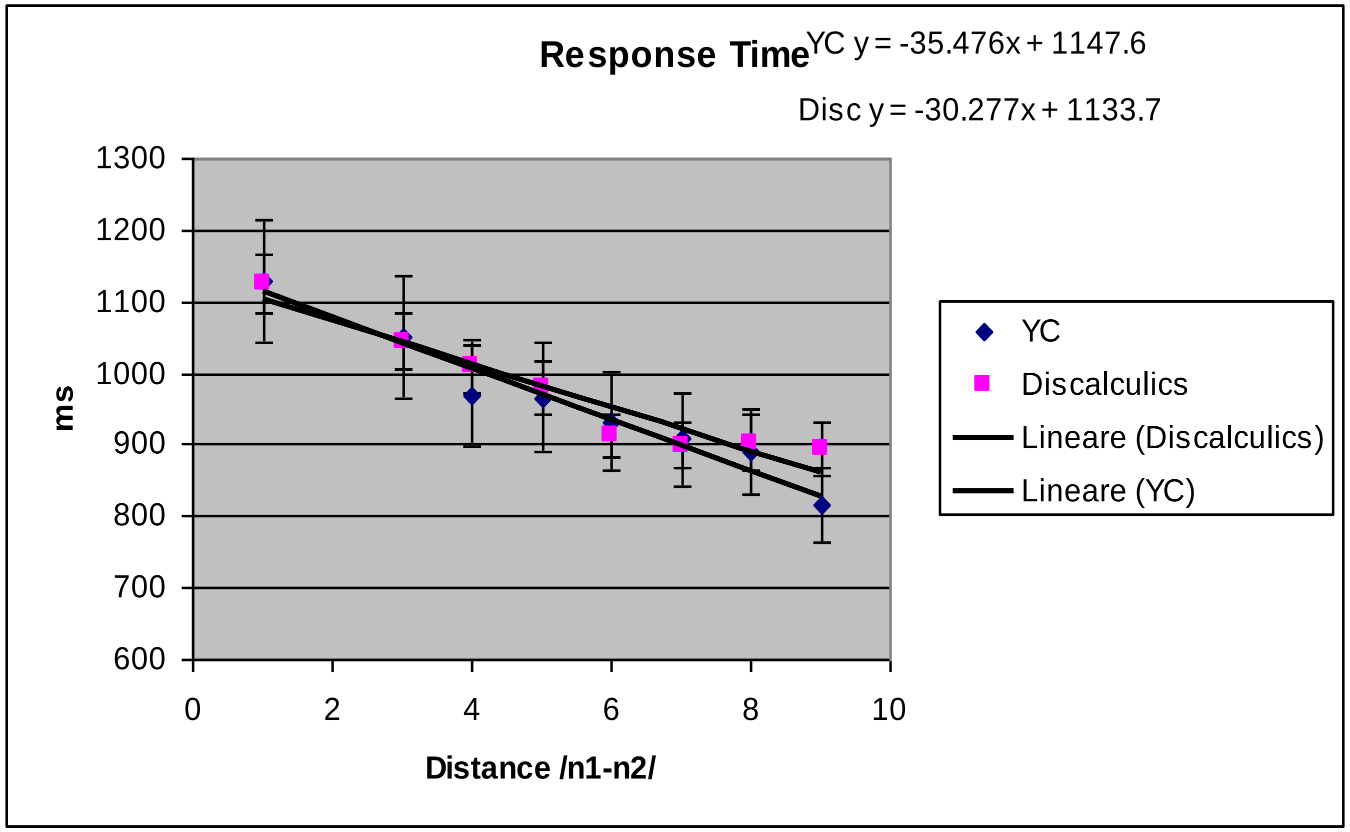
Dyscalculics have a more marked distance effect than age-matched controls

Individual distance effect (beta) correlates with various subtests of the dyscalculia battery, especially semantic tasks and complex mental calculation.

(same for other indices of number comparison – up to r=.6)

Number acuity (w) correlates (0.3) with mean accuracy in symbolic number comparison (N=49, including controls).

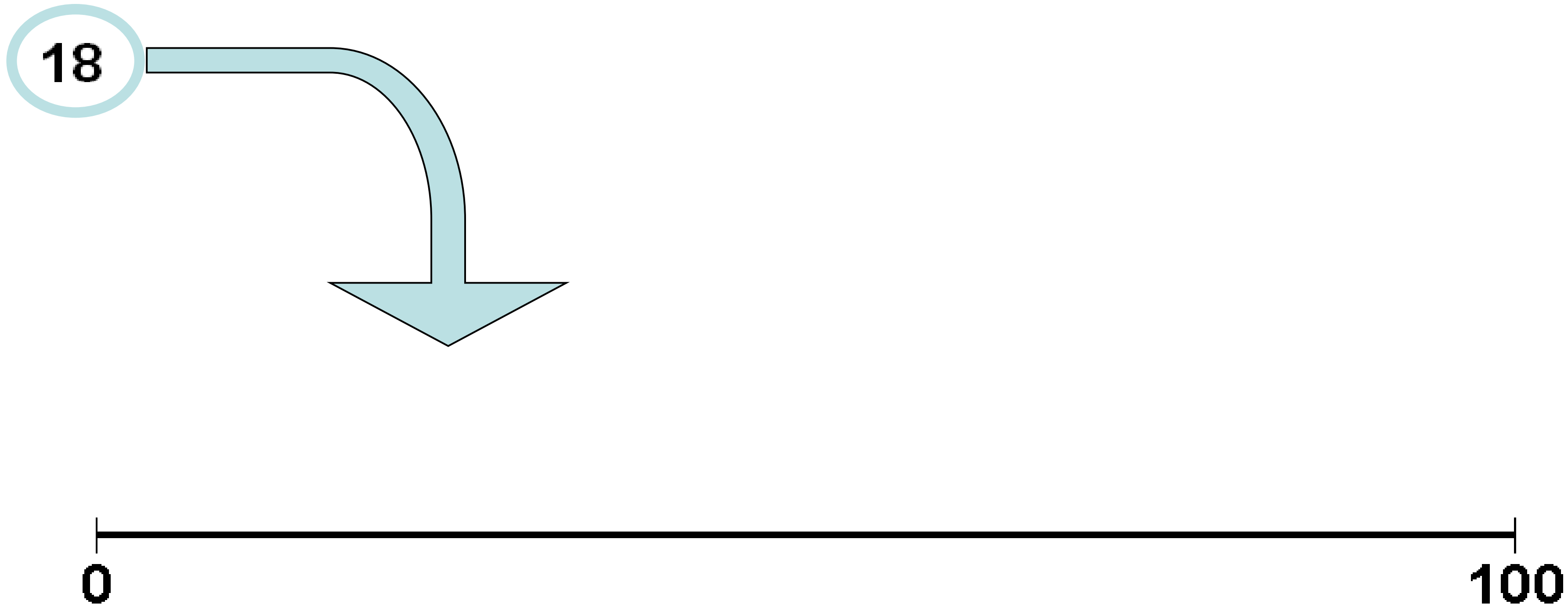




The performance of dyscalculics (10 y.o.) is identical to that of younger controls (7.5 y.o.).

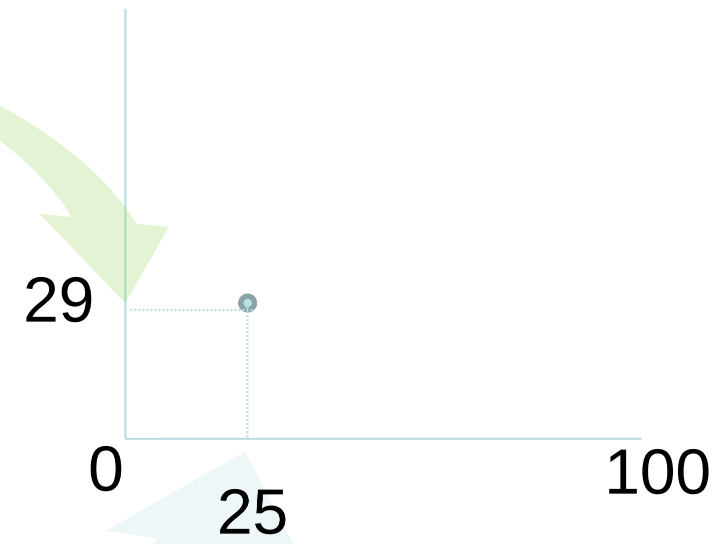
How education shapes the representation of numbers

- The mental representation of numerical magnitude has a spatial nature, similar to a line where numbers are placed from left to right with increasing magnitude (Zorzi et al., 2002, Nature): the **mental number line**
- Children (even preschoolers) can easily indicate how their mental representation of numbers maps onto a physical line. This is diagnostic of the quality of the representation (Siegler & Opfer, 2003, Psych Science)



Number-to-Position task

(Siegler & Opfer, 2003, Psych Science)



$$PE = \left[\frac{|Estimate - Target\ Number\ or\ Quantity|}{scale\ of\ estimation} \right] \cdot 100$$

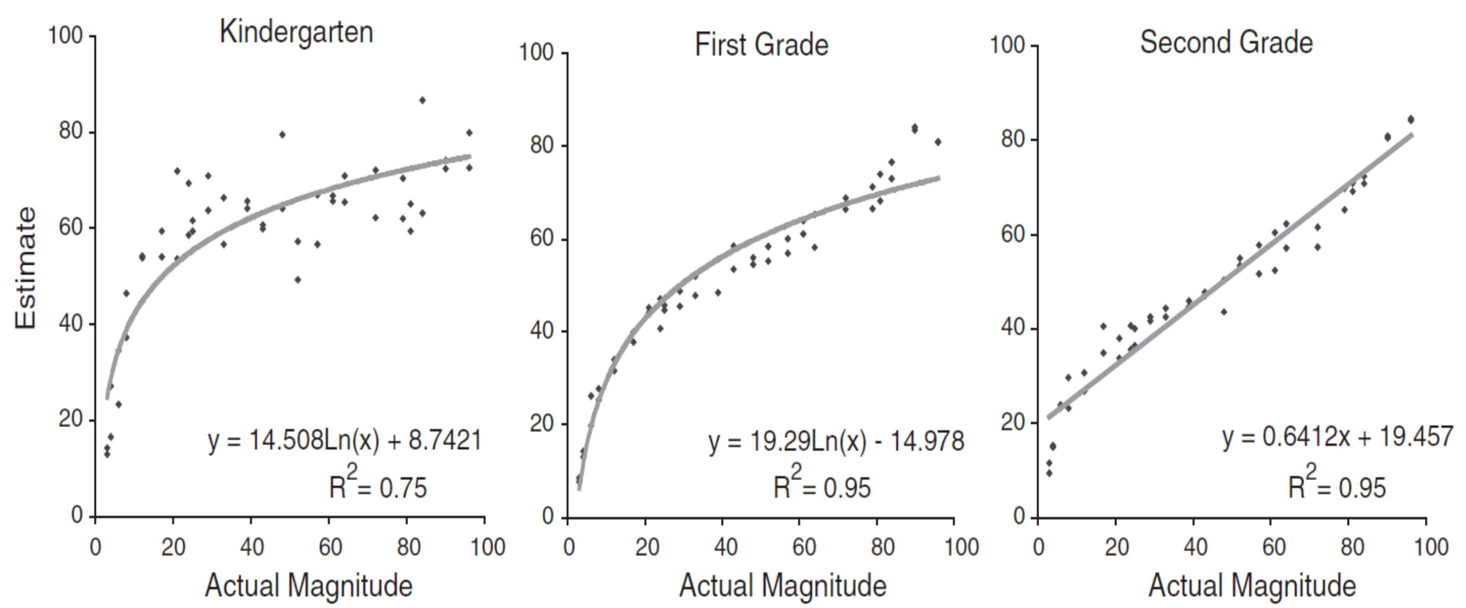
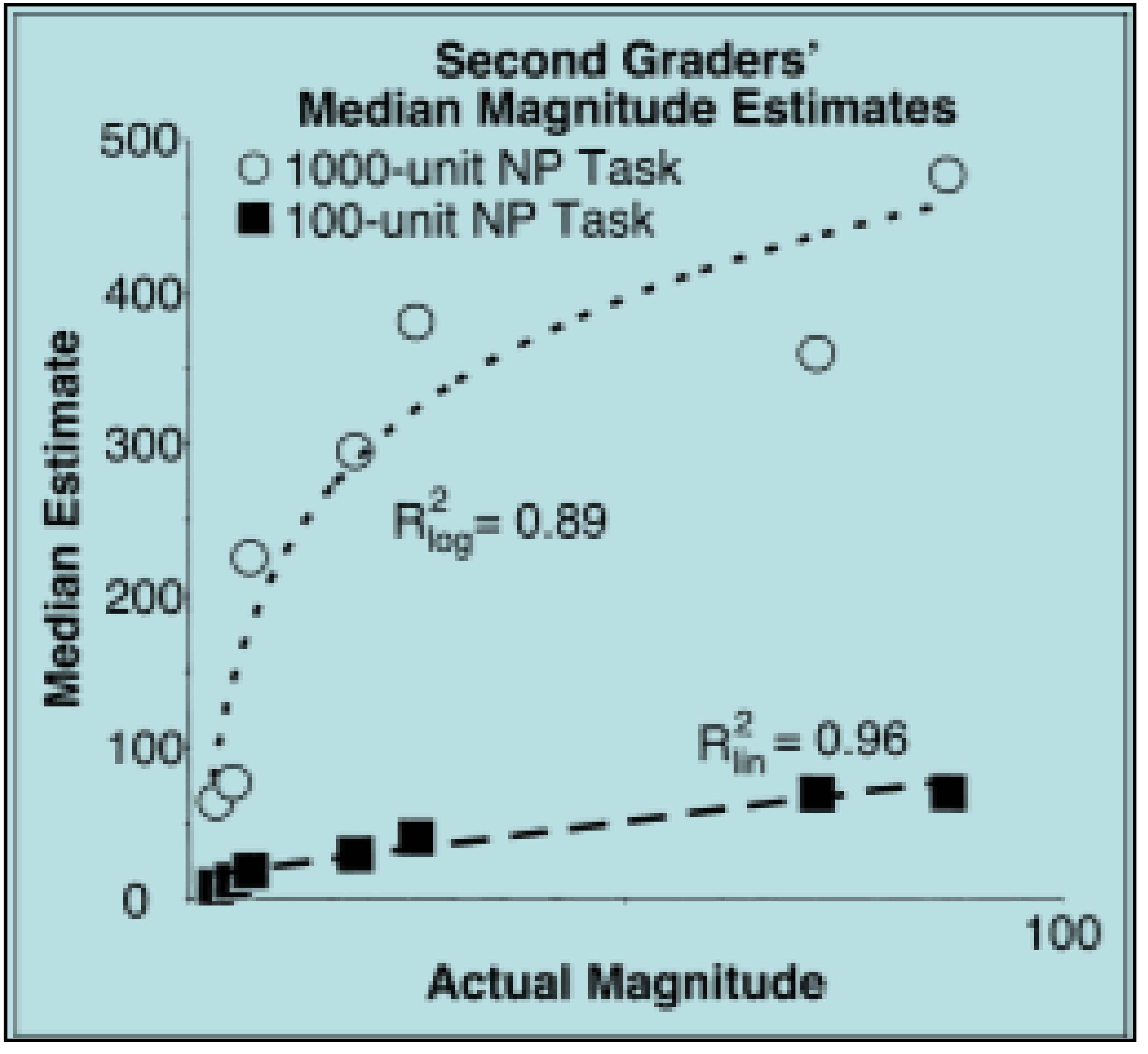


Figure 2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.

Linearity of number line estimates correlates with:

- speed of magnitude comparison (Laski & Siegler, 2007)
- learning of answers to unfamiliar addition problems (Booth & Siegler, 2008)
- overall math achievement test scores (Booth & Siegler, 2006).

A shift from a **Logarithmic** to a **Linear** representation



Co-existence of multiple representations

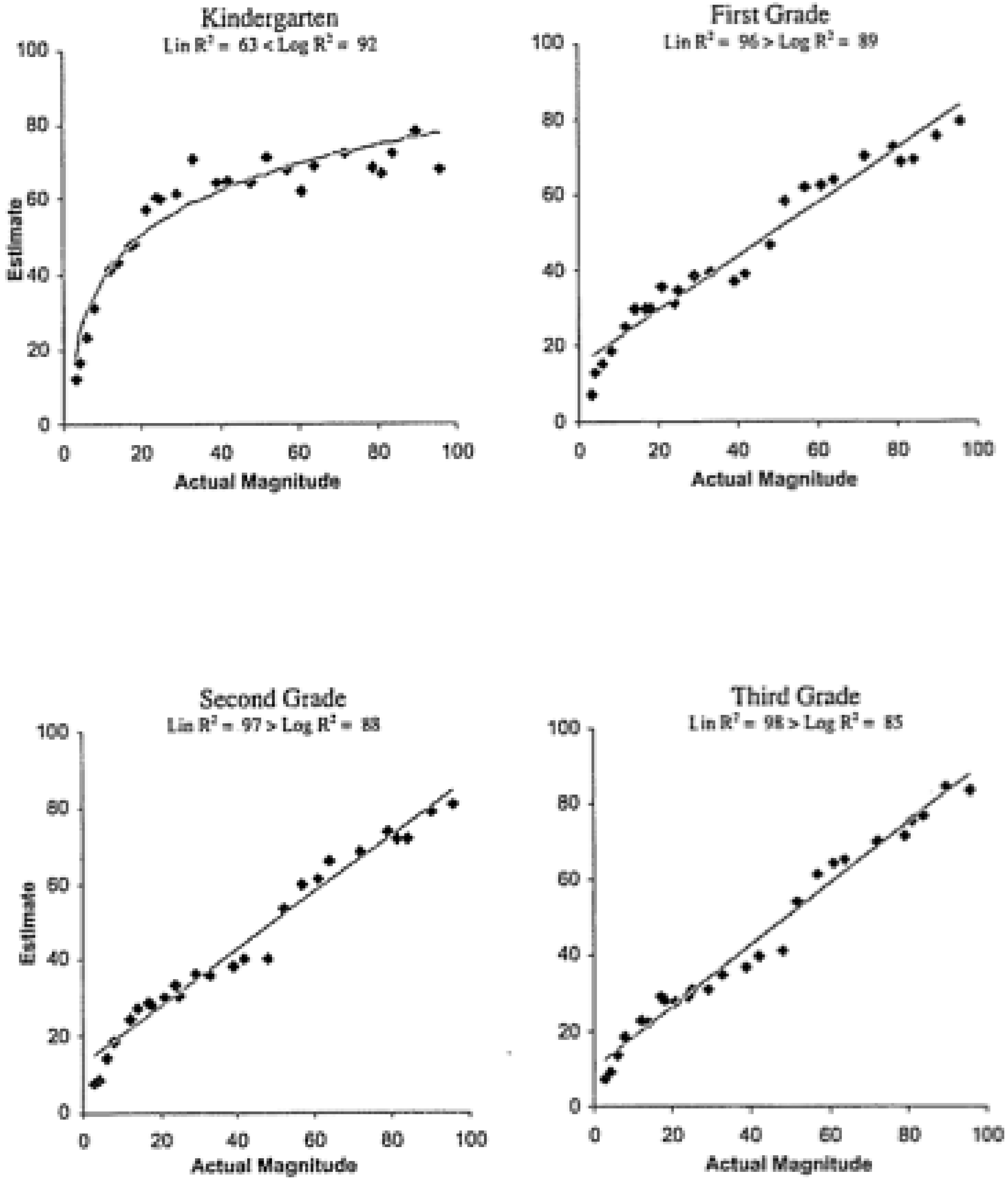


Figure 1. Experiment 1: Best-fitting equations for median number line estimates for kindergartners and first, second, and third graders Lin = linear; Log = logarithmic

Booth and Siegler, 2006

Numerical estimation in preschoolers (4-6 y.o.)

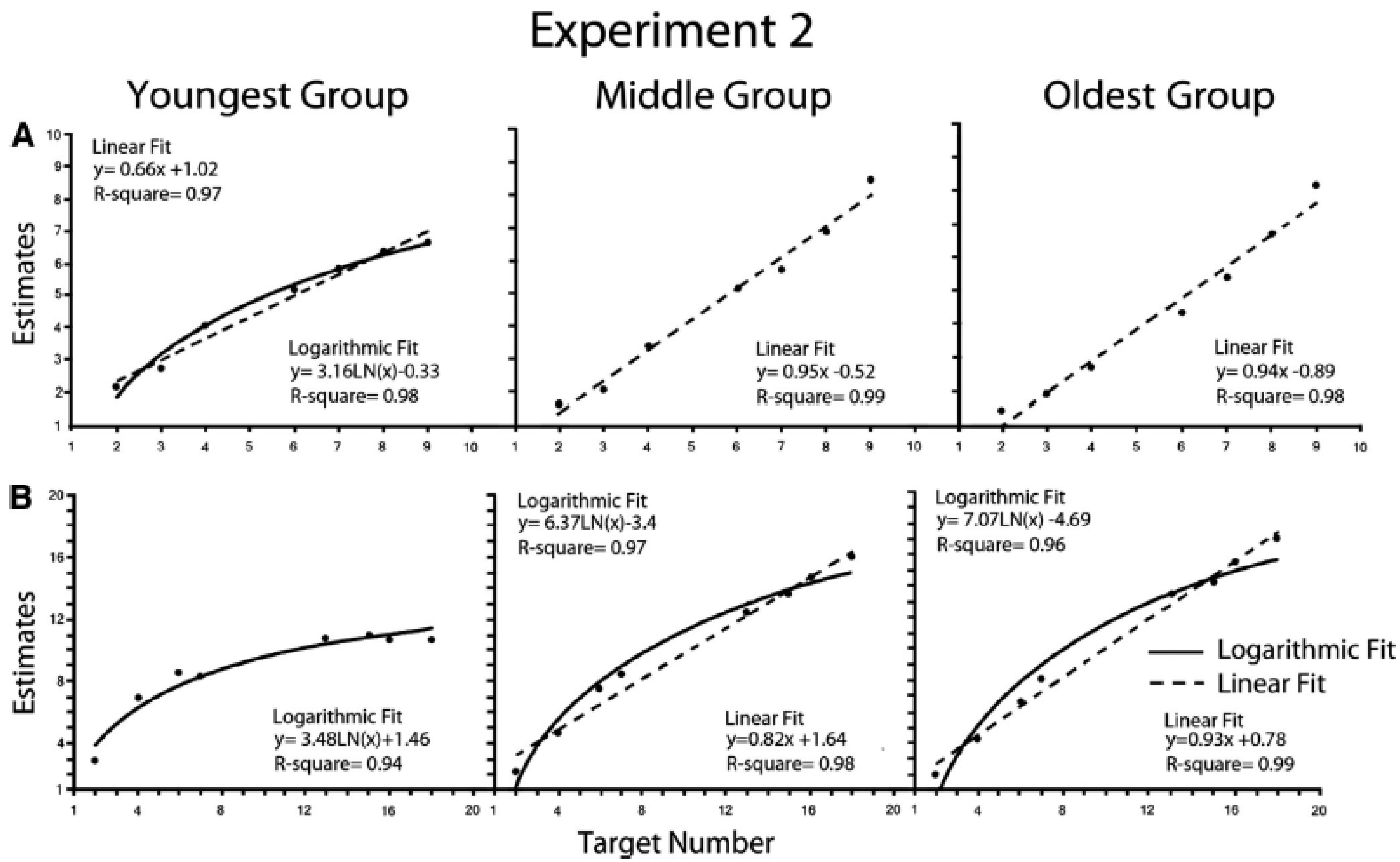


Figure 2. Children estimates and best fitting models in Experiment 2 as a function of age group for 1–10 (A) and 1–20 (B) number lines.

(data from 373 Italian children)

Same developmental pattern in a much smaller number range for preschool children

Berteletti et al., 2010, *Dev. Psych.*

Numerical estimation in dyscalculic children

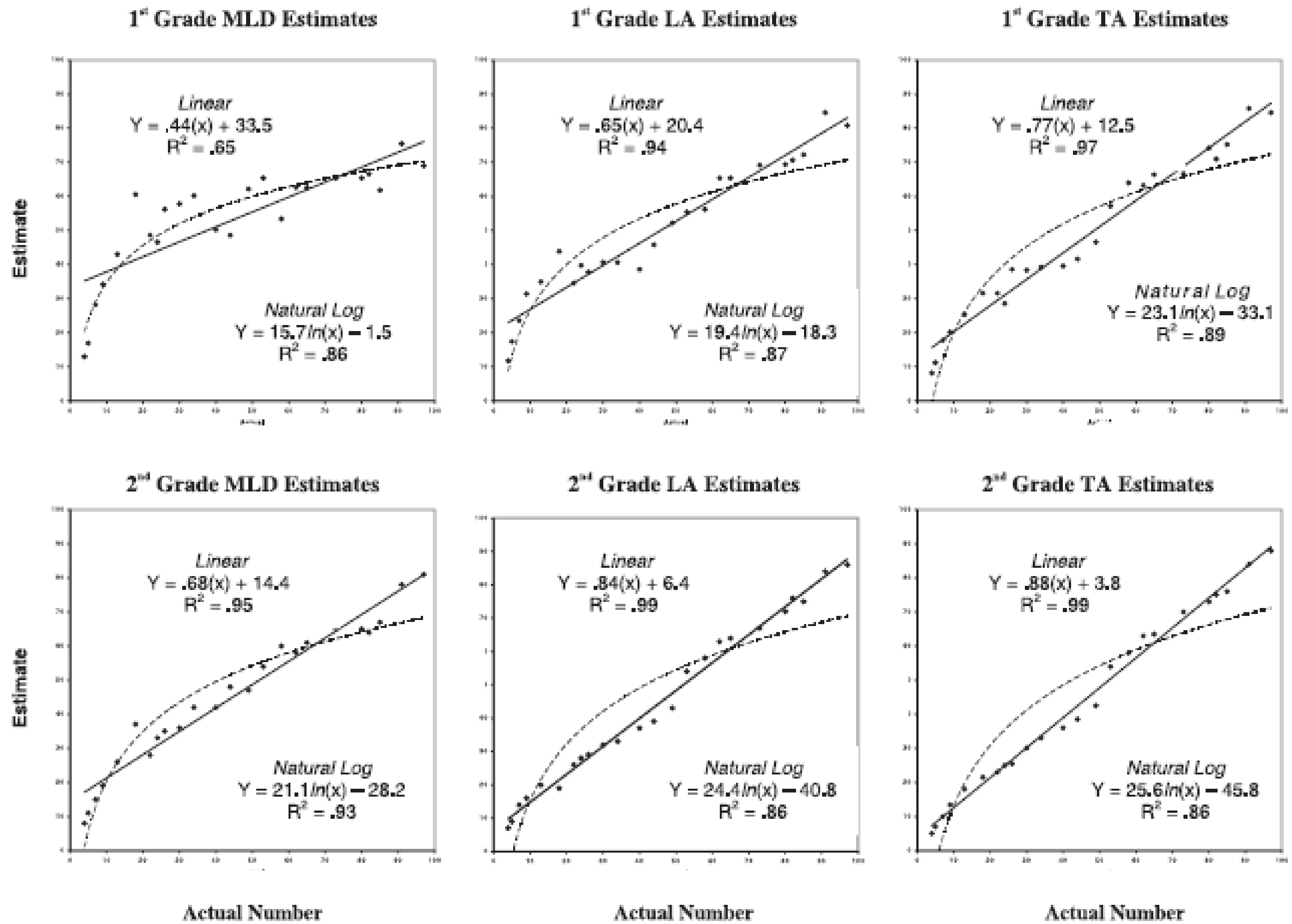


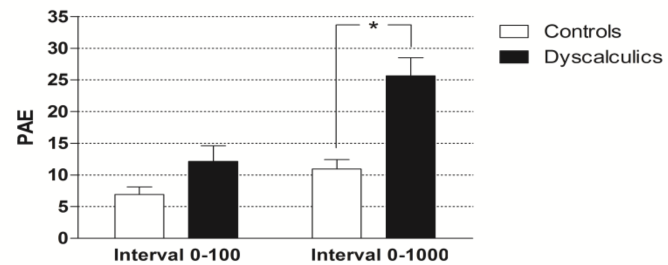
FIGURE 1 Linear and logarithmic fits to median number line placements in 1st (top) and 2nd (bottom) grades. MLD = math disabled, LA = low achieving, and TA = typically achieving.

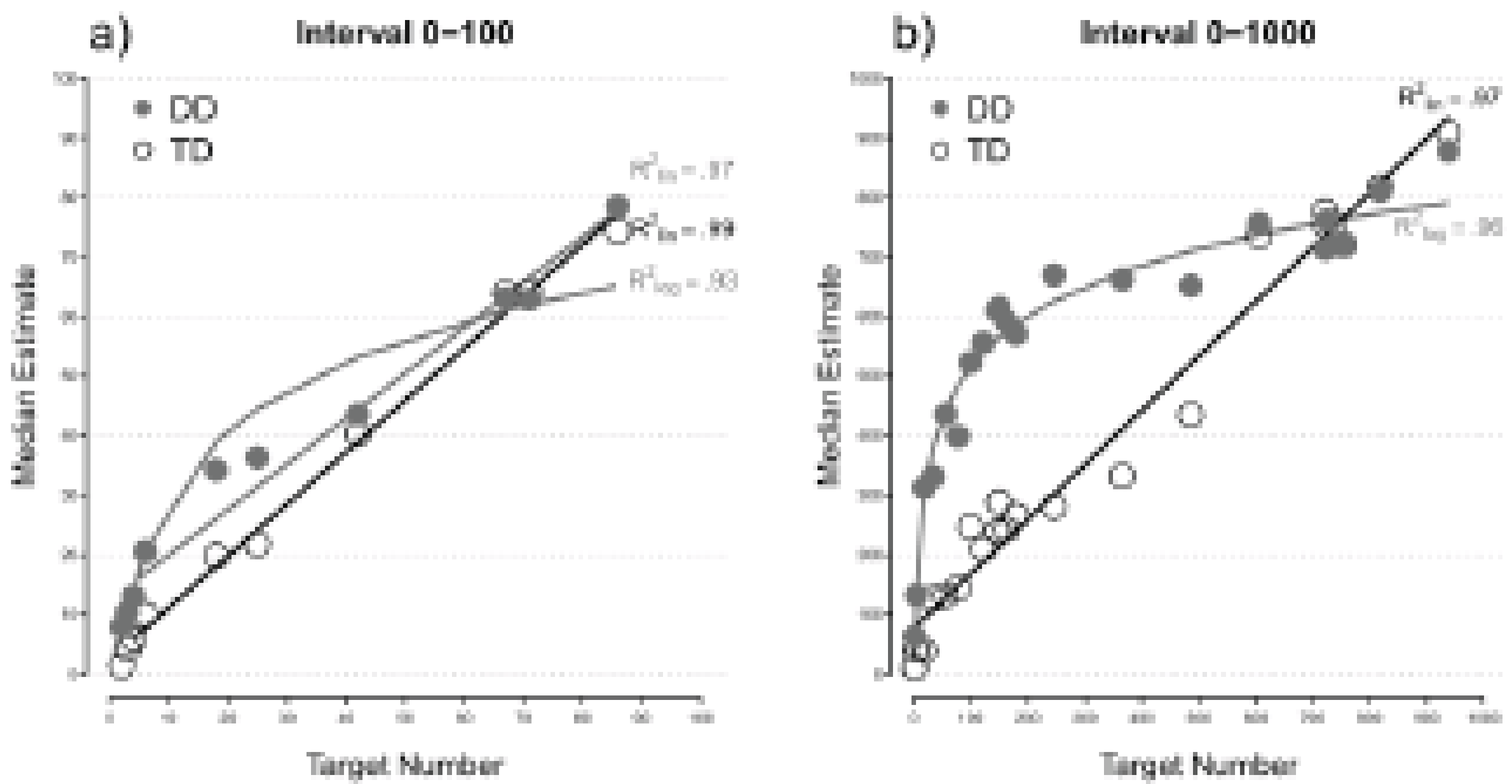
Mapping is imprecise in dyscalculic (MLD) children and can still be logarithmic compared to controls

Geary et al., 2008, *Dev. Neuropsych.*

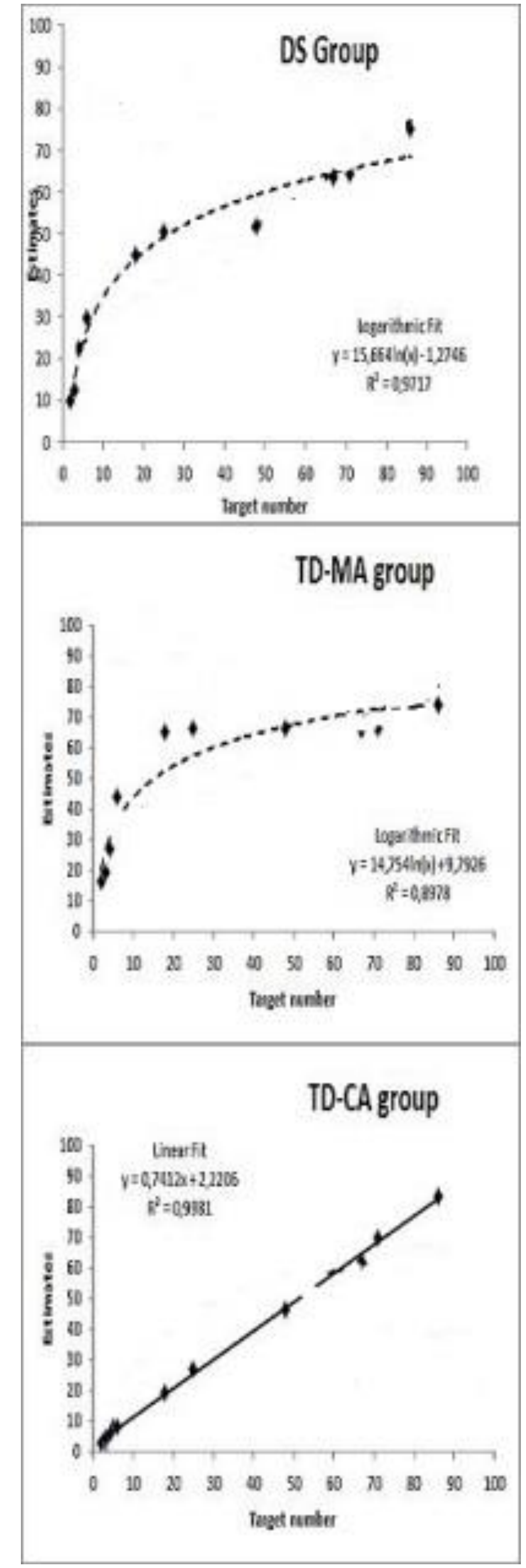
Numerical estimation as a marker of math learning difficulties

Estimation pattern in children with math difficulties is still logarithmic when their peers show linear positioning





Sella et al., 2013, *Learn. Disab.*



Numerical estimation in Down Syndrome
Lanfranchi, .., Zorzi, 2014, *Res. Dev. Disab.*

Interim summary (number-to-position task)

The mapping before formal teaching is logarithmic/intuitive

Developmental pattern: mandatory logarithmic phase followed by linear mapping

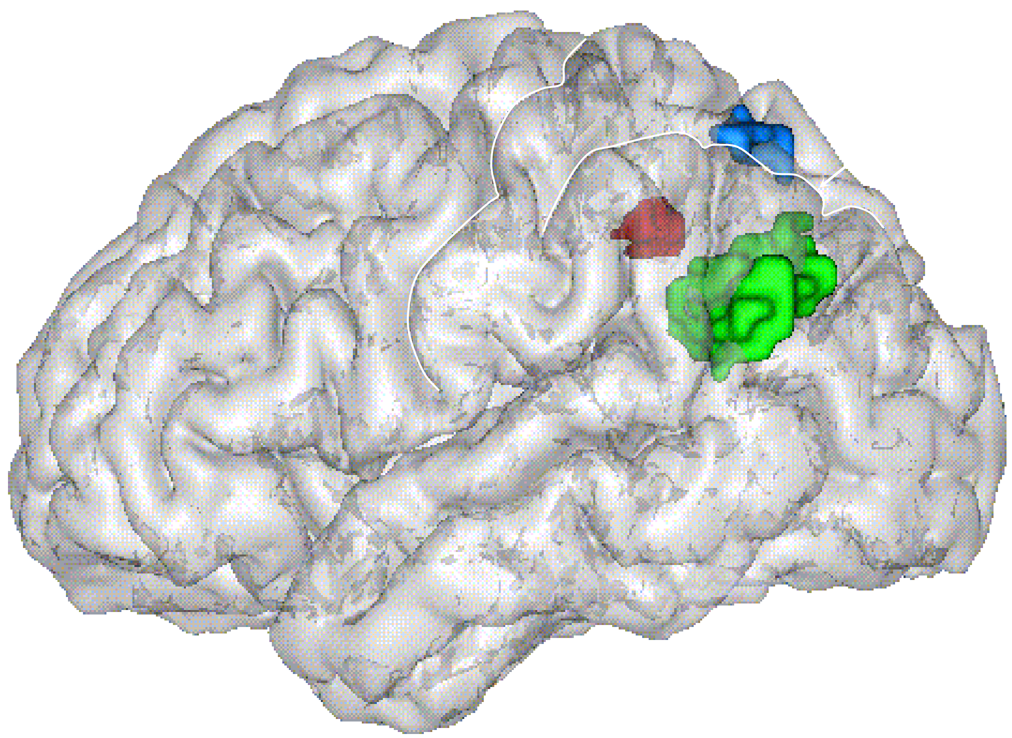
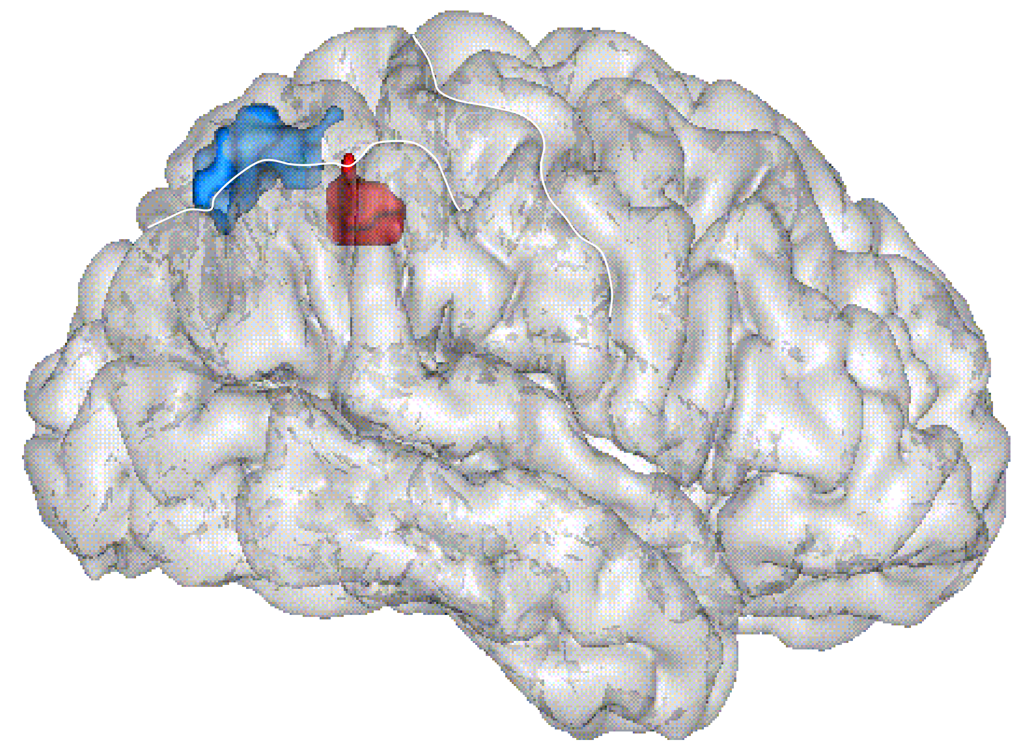
Shift between representations, but multiple representations (for different number scales) may co-exist early on

Linearity correlates with math achievement

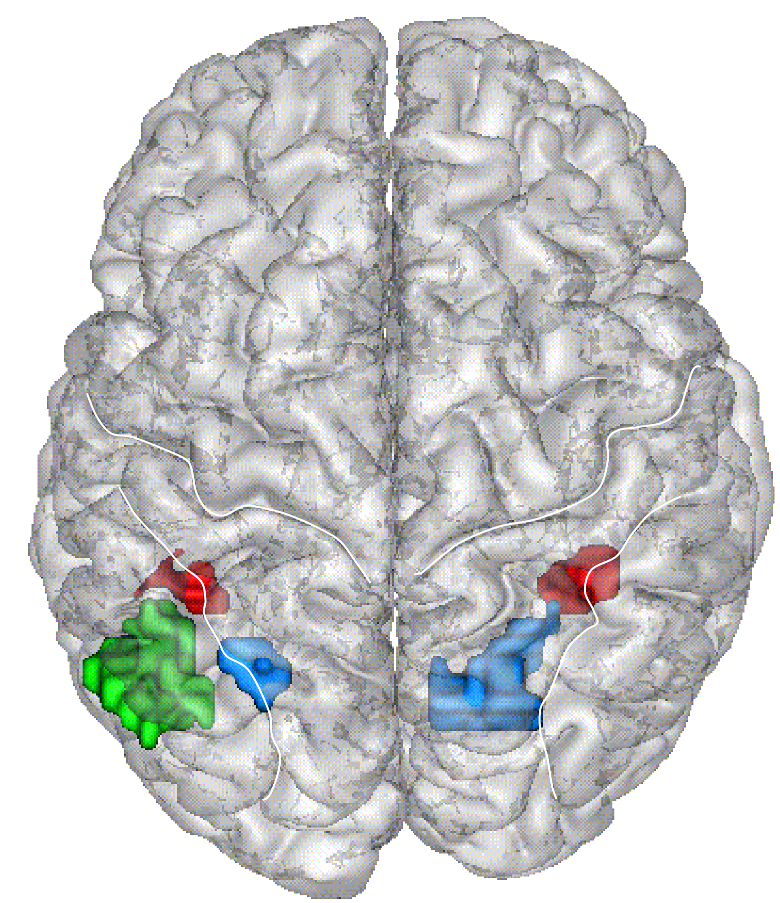
The mapping showed by dyscalculic children is less precise and can be logarithmic on a scale in which their peers are linear

Neural bases of numerical cognition and dyscalculia

Parietal circuits for number processing

1. Bilateral intraparietal system – HIPS: core quantity system
2. Left angular gyrus: verbal processing of numbers
3. Posterior superior parietal system: spatial and nonspatial attention involved in number processing



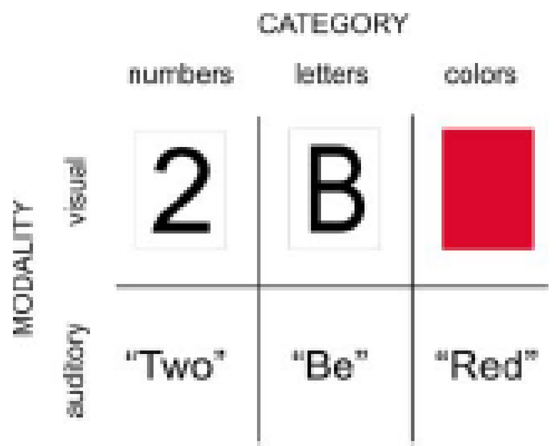
bilateral horizontal segment of intraparietal sulcus (HIPS)

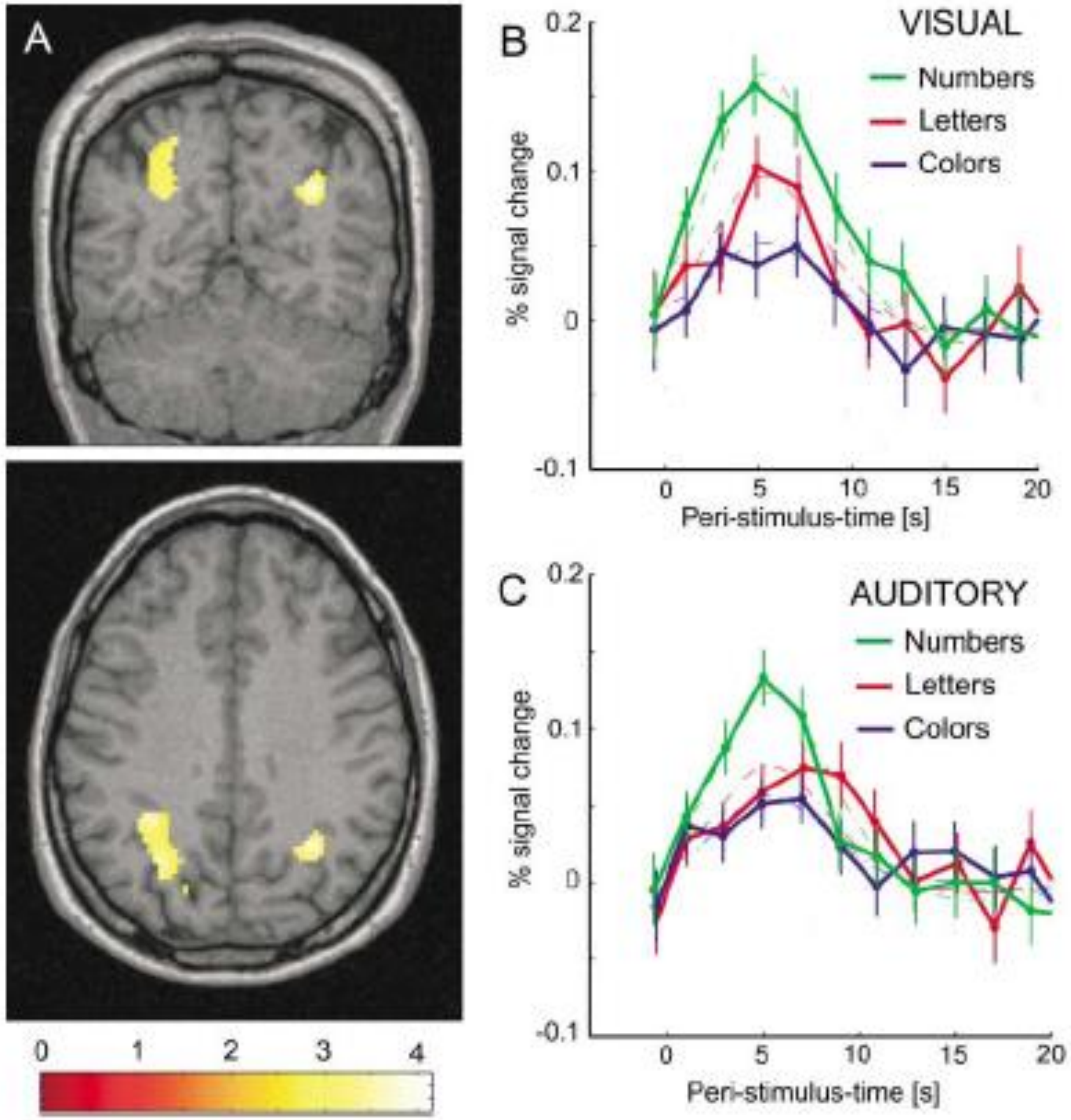
left angular gyrus (AG)

bilateral posterior superior parietal lobe (PSPL)

Dehaene et al., 2003, *Cognitive Neuropsychology*

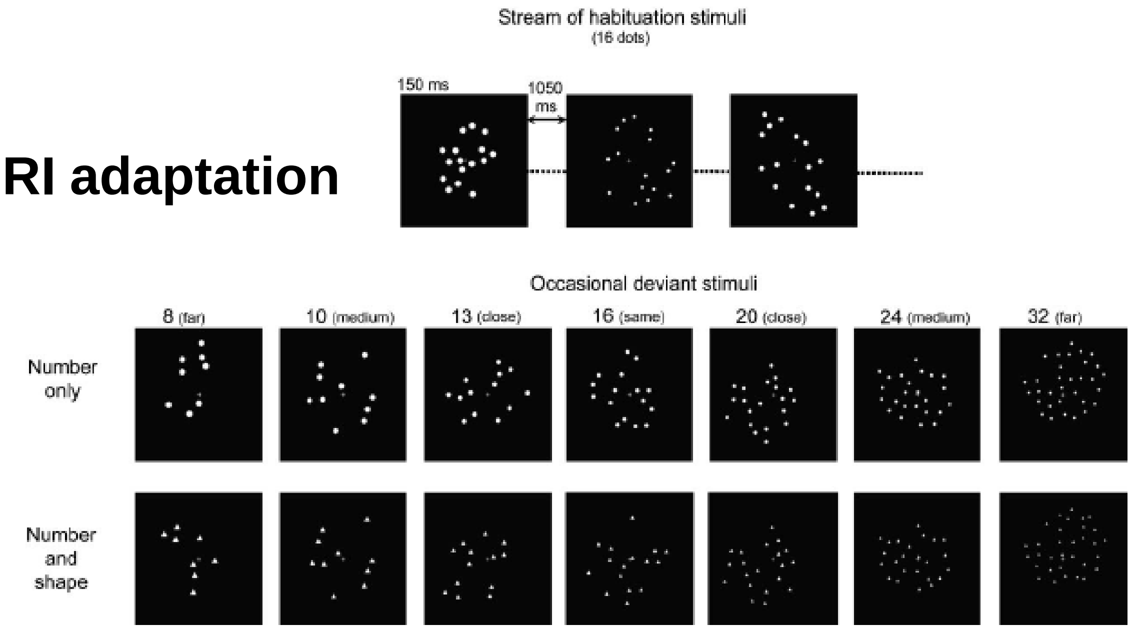
A supra-modal representation of numbers in IPS

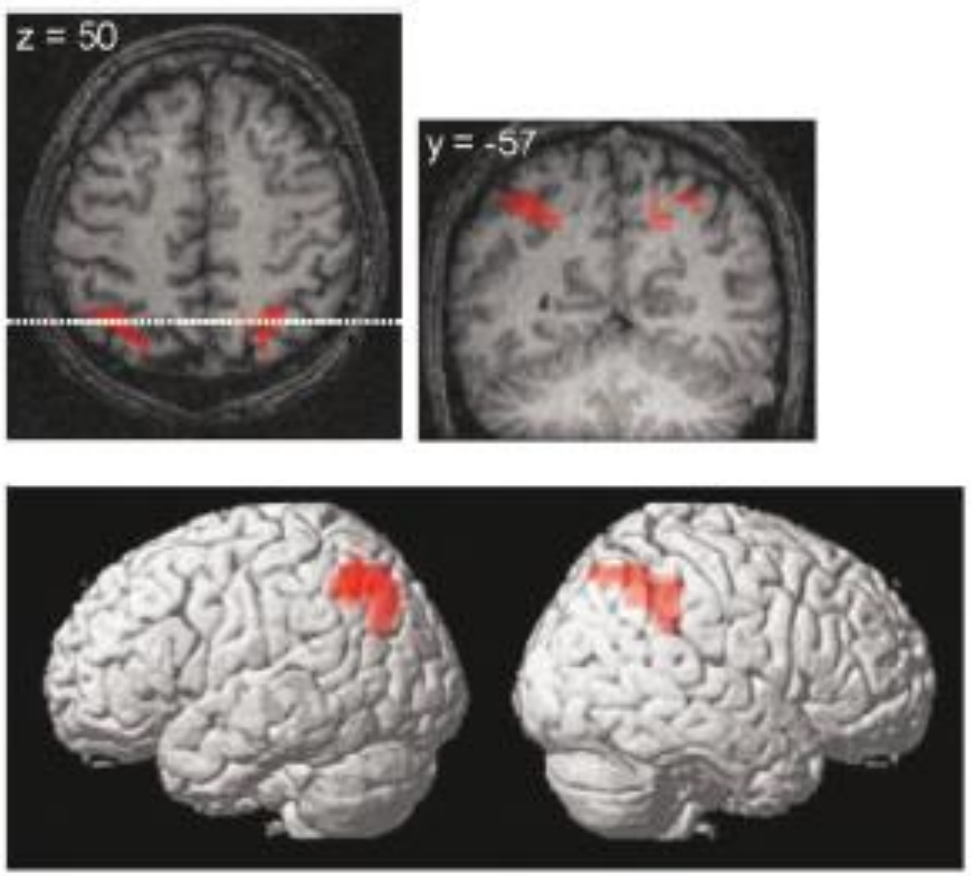


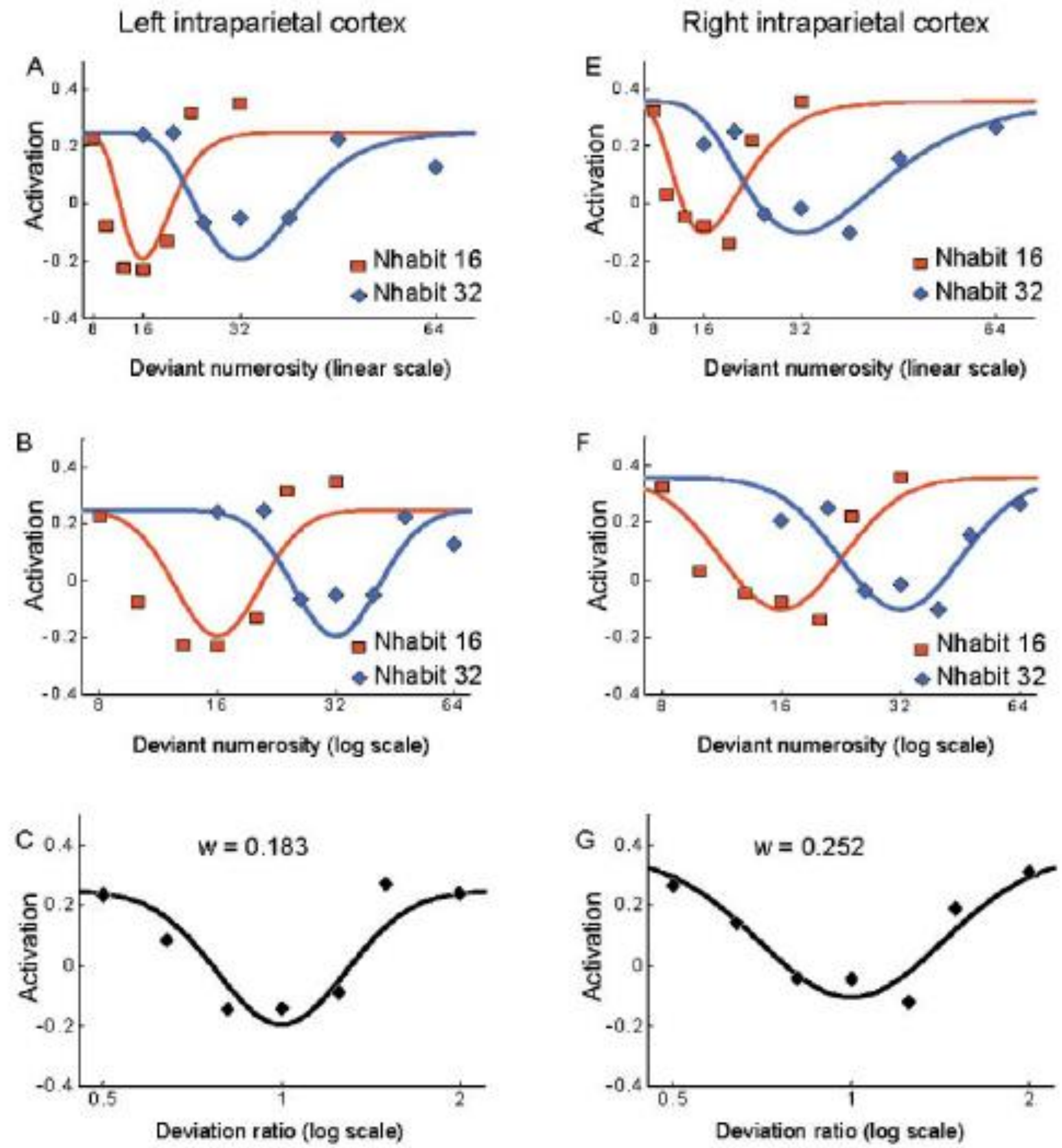


IPS responds to non-symbolic numbers (numerosities

fMRI adaptation

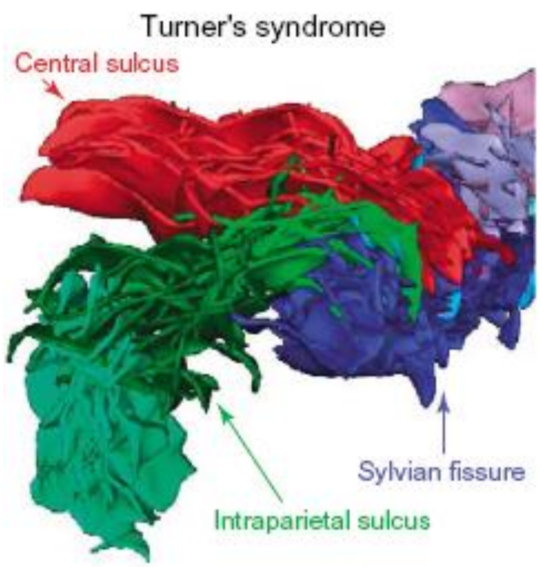


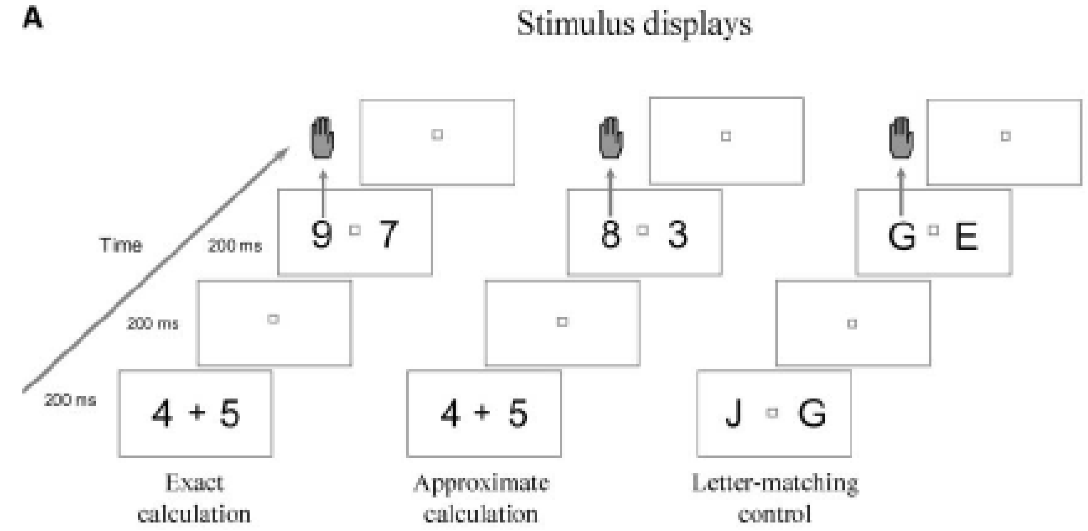


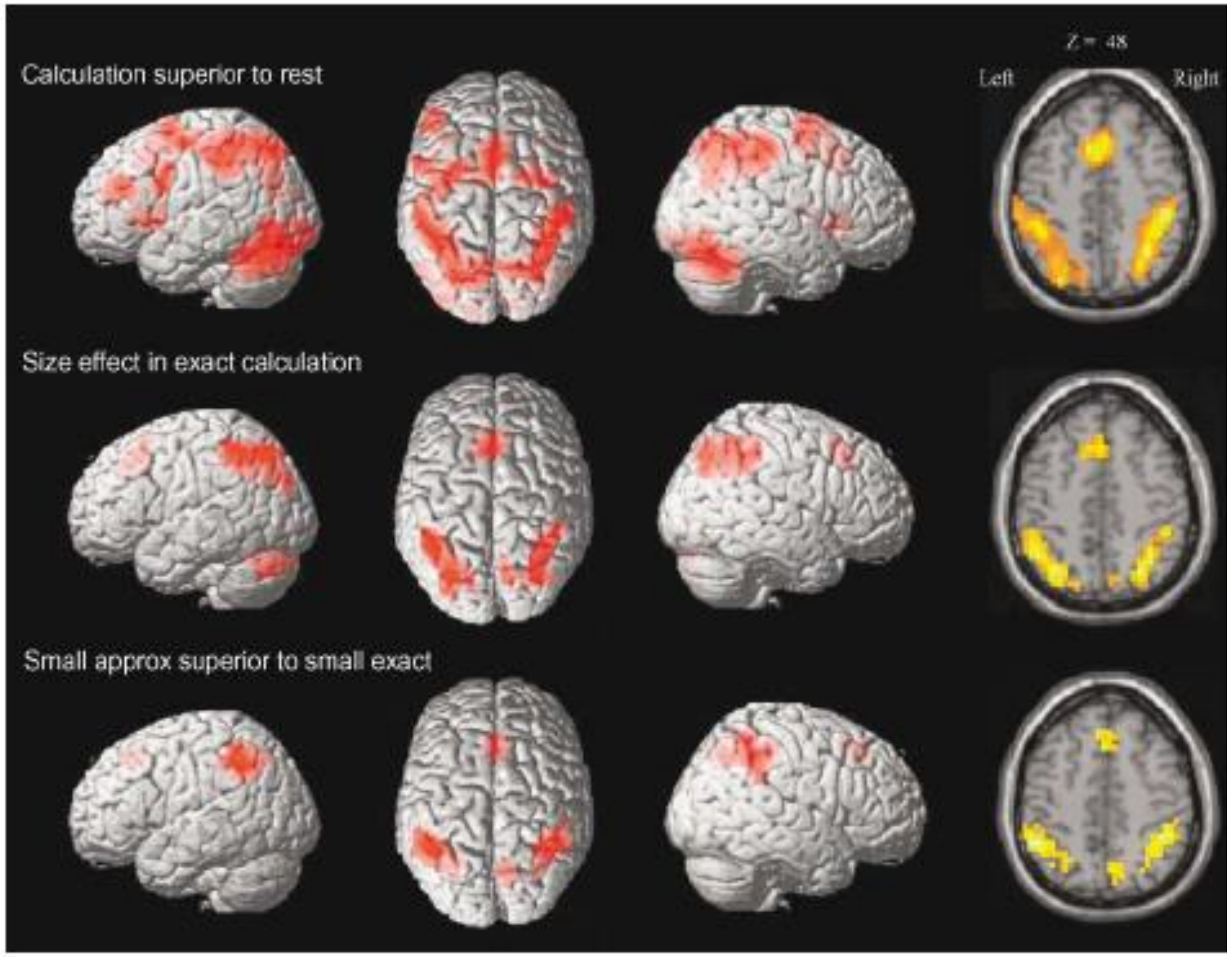


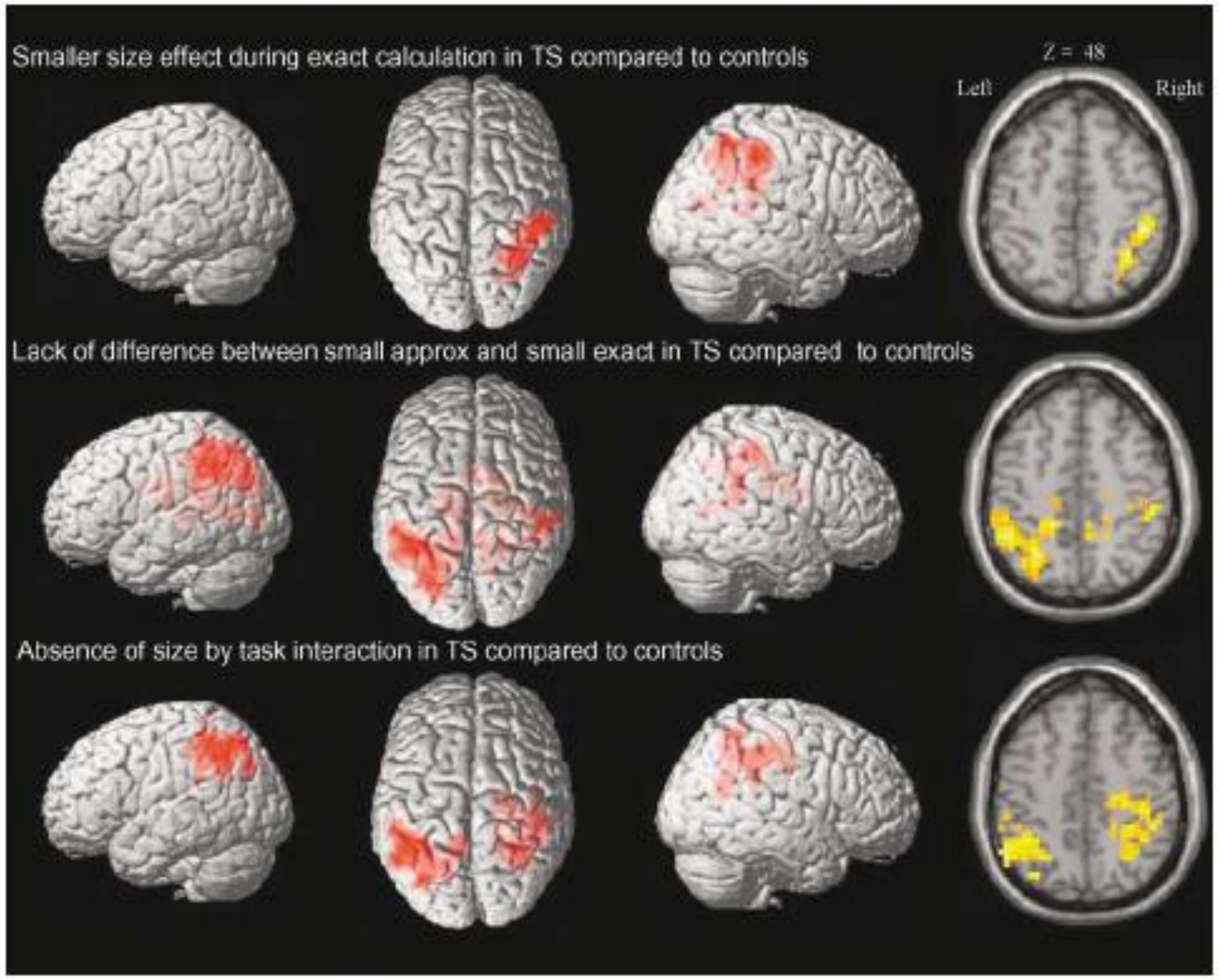
Piazza et al., 2004, *Neuron*

Functional and structural abnormalities of IPS in dyscalculia







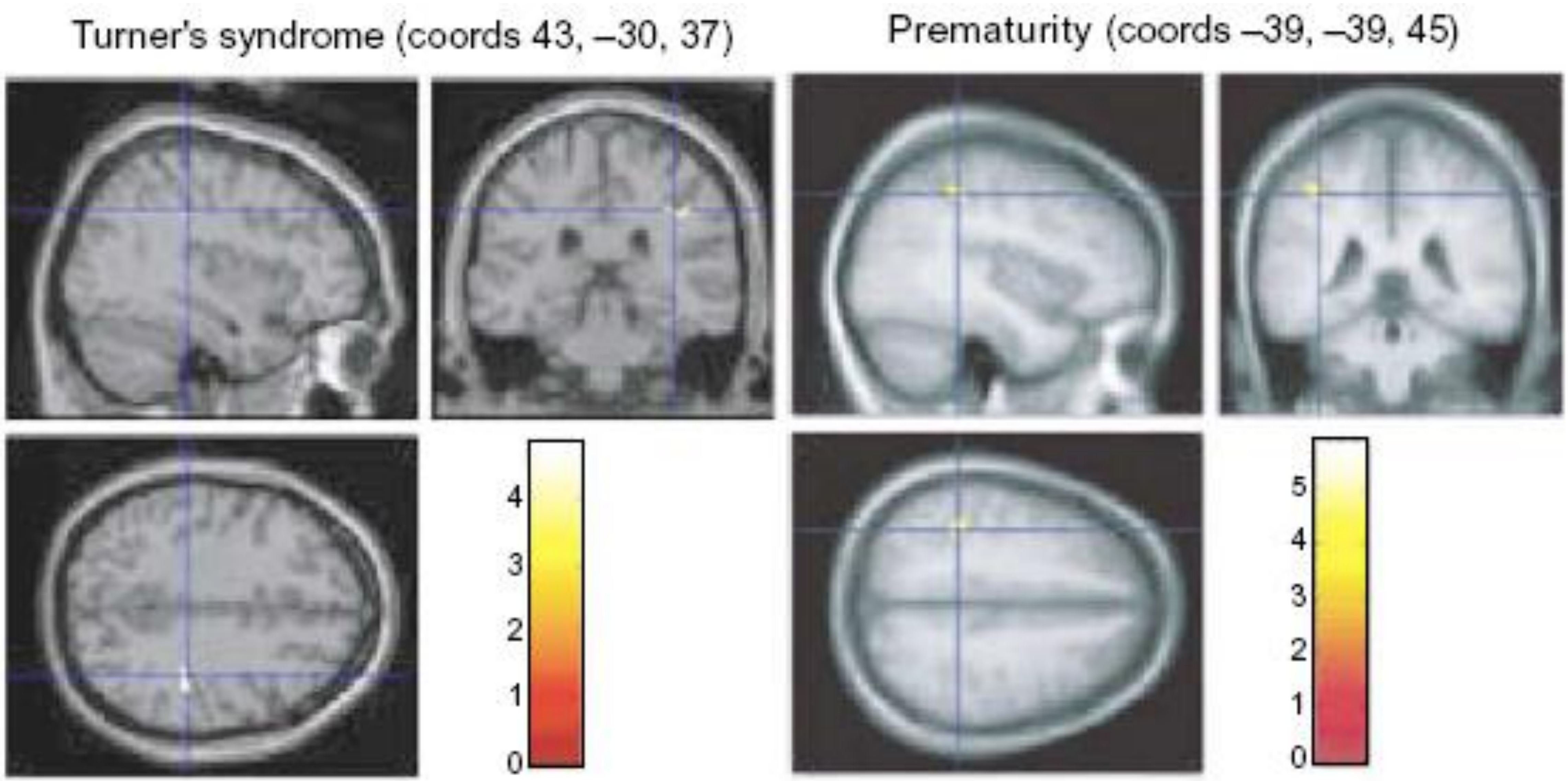


Controls

Dyscalculics (Turner)

Molko et al., 2003, *Neuron*

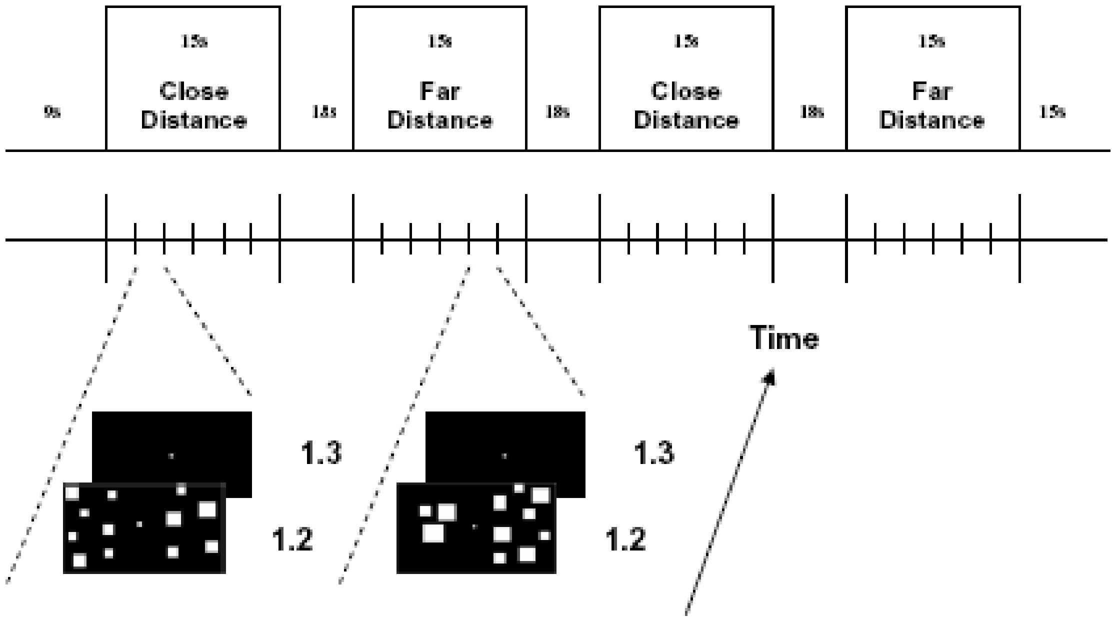
Voxel Based Morphometry (VBM) in dyscalculia



VBM shows a decrease of gray matter density in IPS

(Turner: Molko et al., 2003, Neuron)
(Very-low birth-weight: Isaacs et al., 2001, Brain)

Dysfunction of right IPS during non-symbolic number comparison



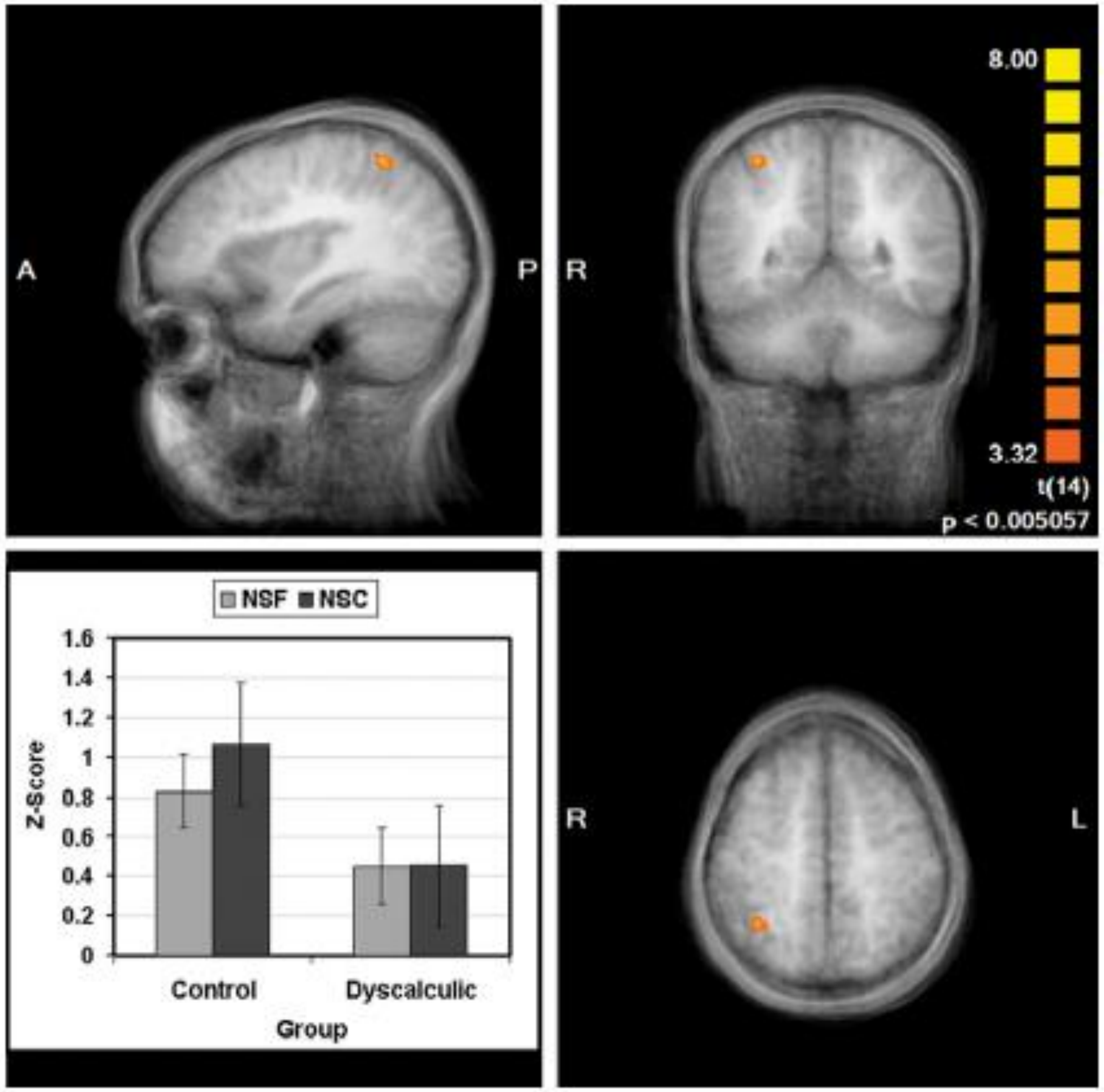
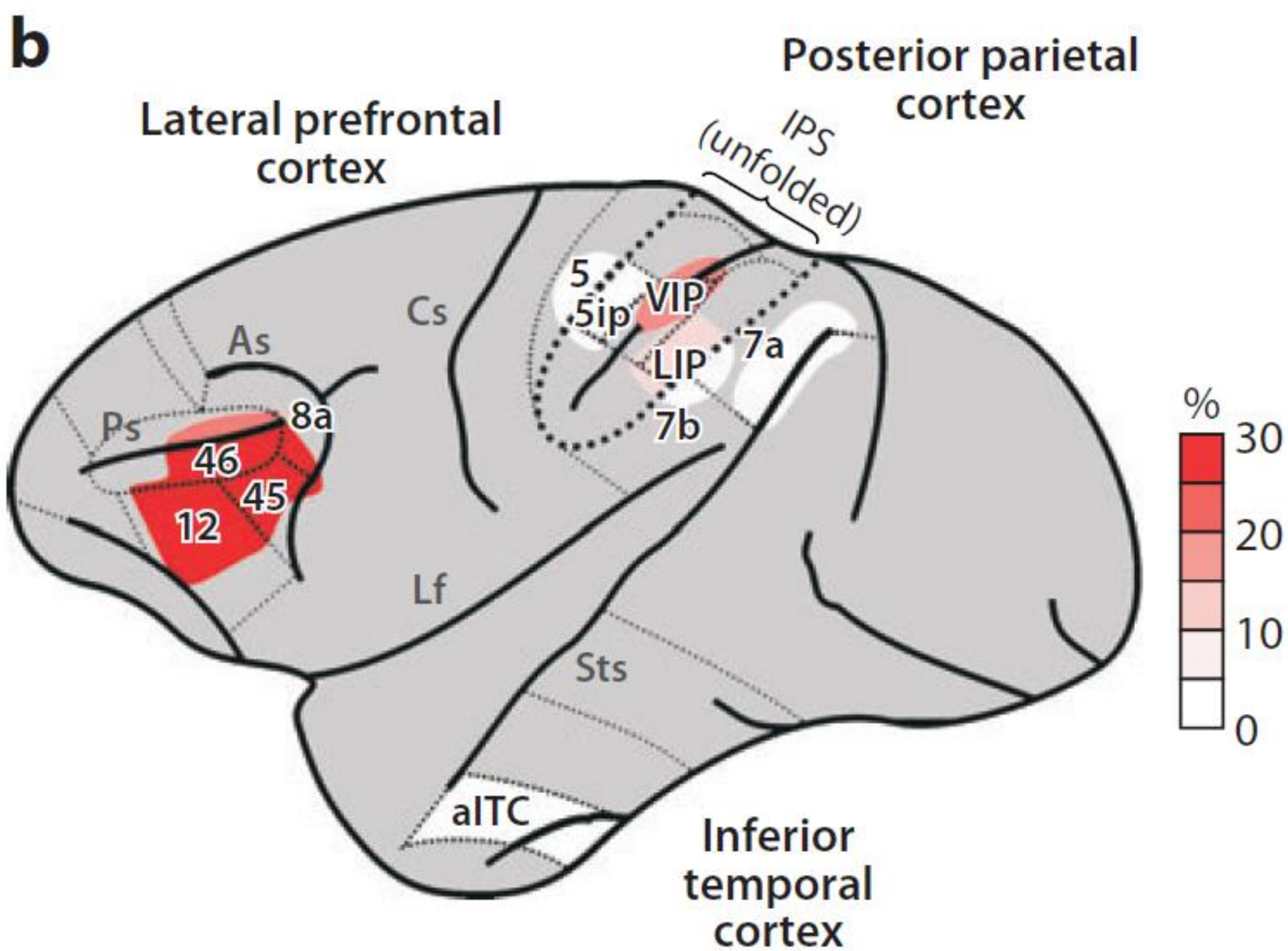
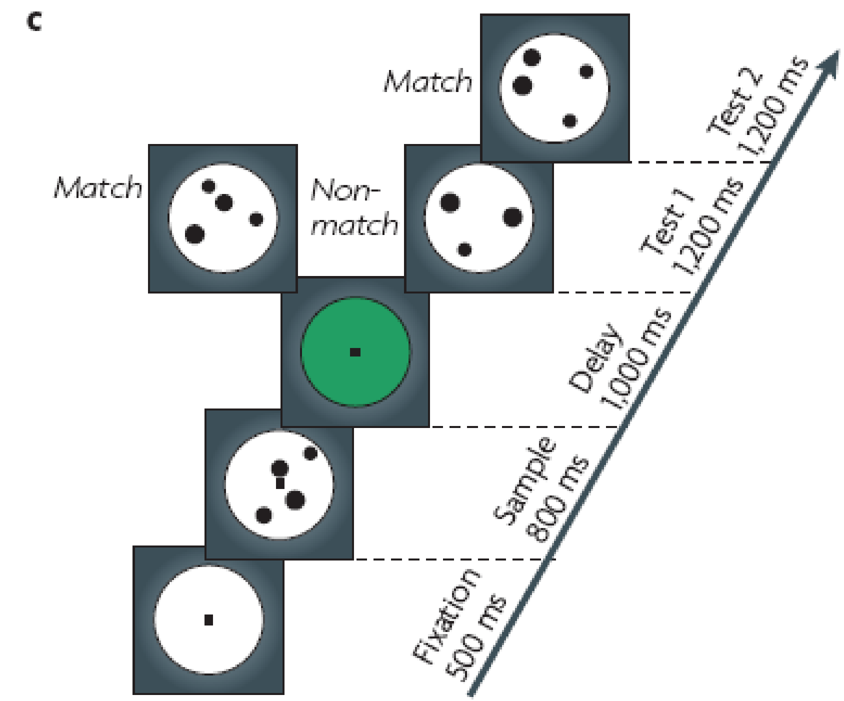


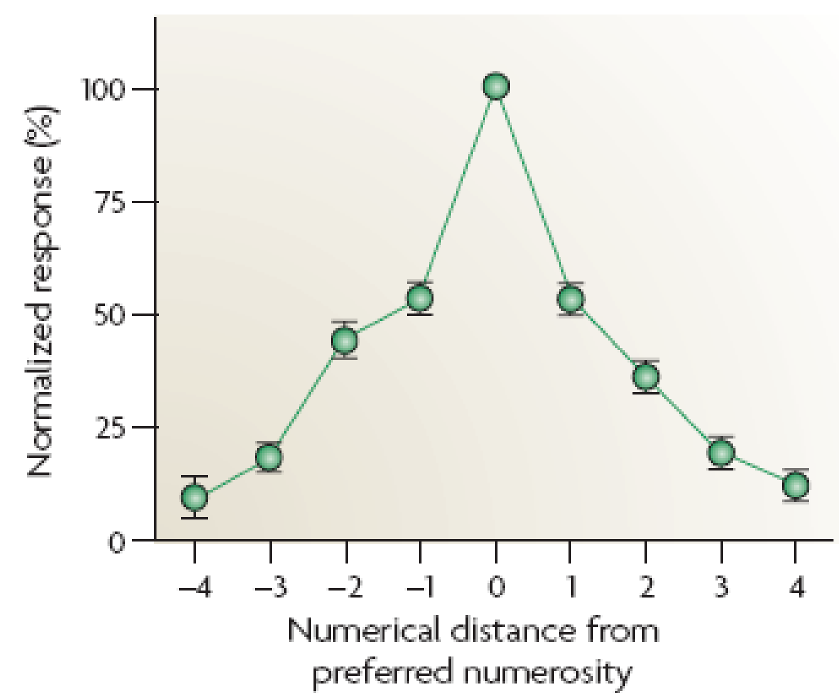
Figure 1. Interaction of group X distance in the right intraparietal sulcus.
Statistical map showing interaction of group x distance ($p < 0.05$, corrected at the cluster level) overlaid on an average image of all participants' high-resolution structural MRI scans level. The bar chart shows the parameter estimates for this region for both groups. Error bars represent the standard error of the mean. (NSC, Nonsymbolic Close Distance; NSF. Nonsymbolic Far Distance.)

Price et al., 2008, *Current Biology*

Number neurons” in the monkey brain







Nieder, 2005, *Nature Reviews Neuroscience*
Nieder & Dehane, 2009, *Ann. Rev. Neuroscience*

Conclusions

1. Complex numerical abilities and mathematical learning are built upon a philogenetically ancient “number sense” that allows us to perceive and represent numerical magnitude.

2. Dyscalculic children show a core deficit in number sense. Number acuity is severely impaired and the subitizing range is reduced.

3. A structural abnormality of IPS might be a key neurofunctional marker of dyscalculia

4. Early screening should be based on non-symbolic and non-verbal tasks that are independent from math achievement / learning outcome.