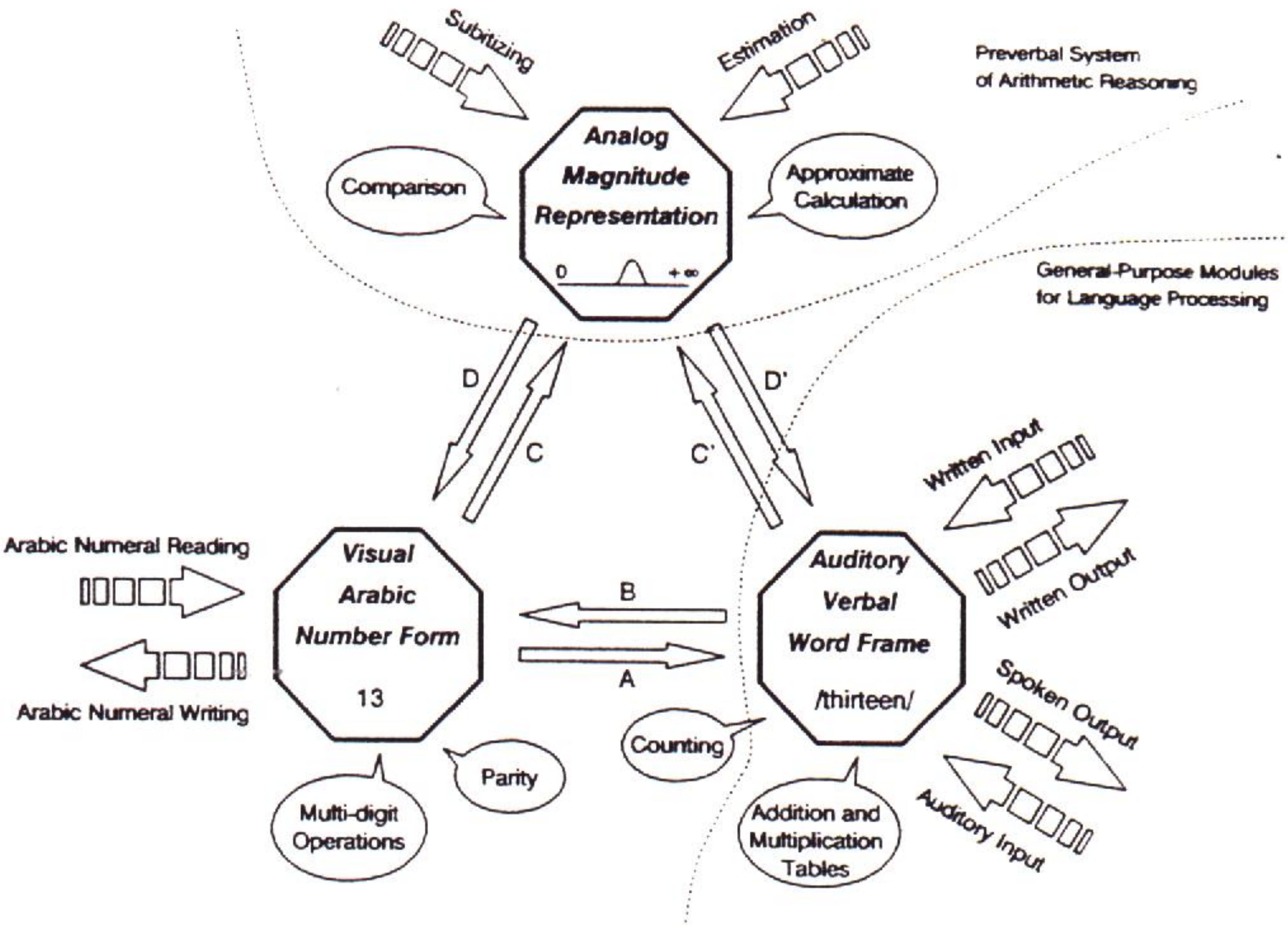
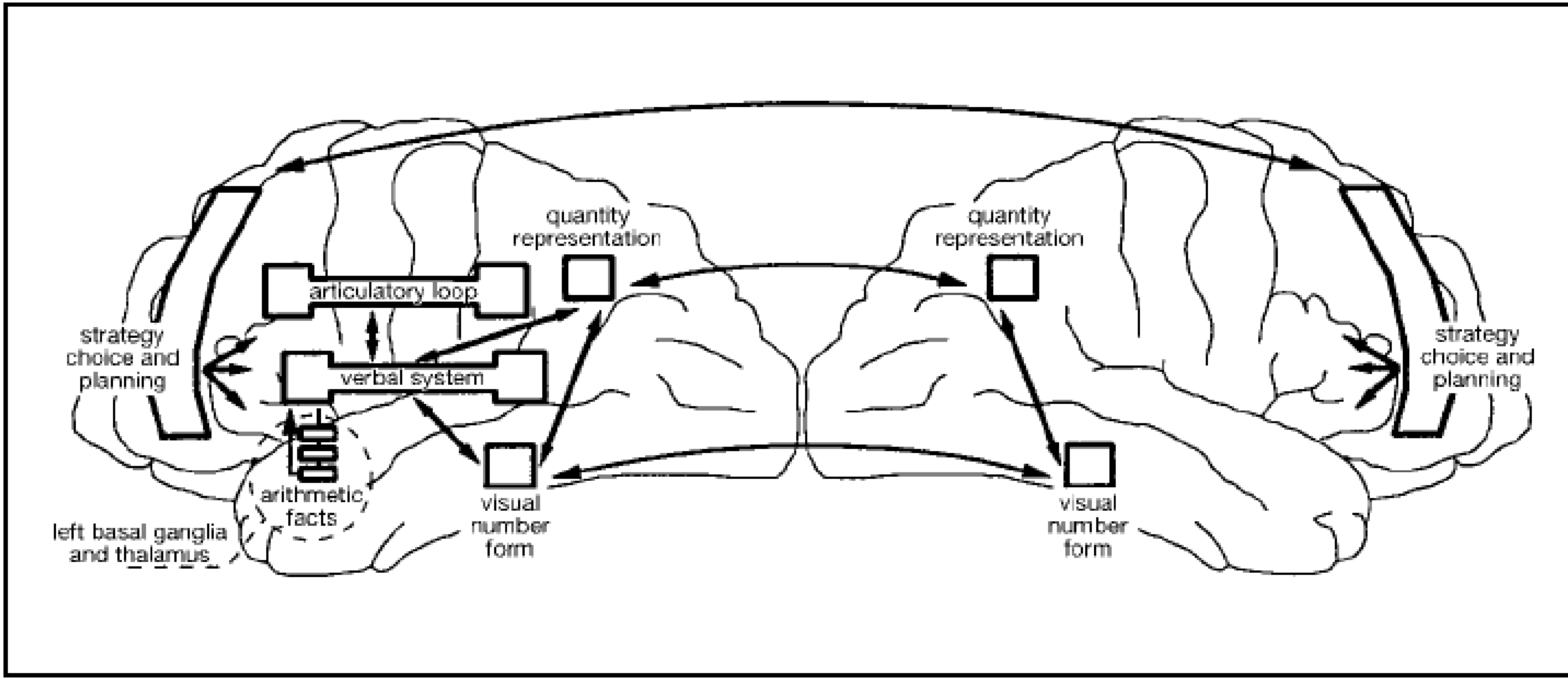
Prof. Marco Zorzi
University of Padova

Numerical cognition and dyscalculia (part 1)

Triple-code model of numerical cognition



The triple-code model (Dehaene, 1995) is the leading cognitive model of number processing and calculation. It is the reference model for many neuropsychological testing batteries



Triple-code model and its neural bases

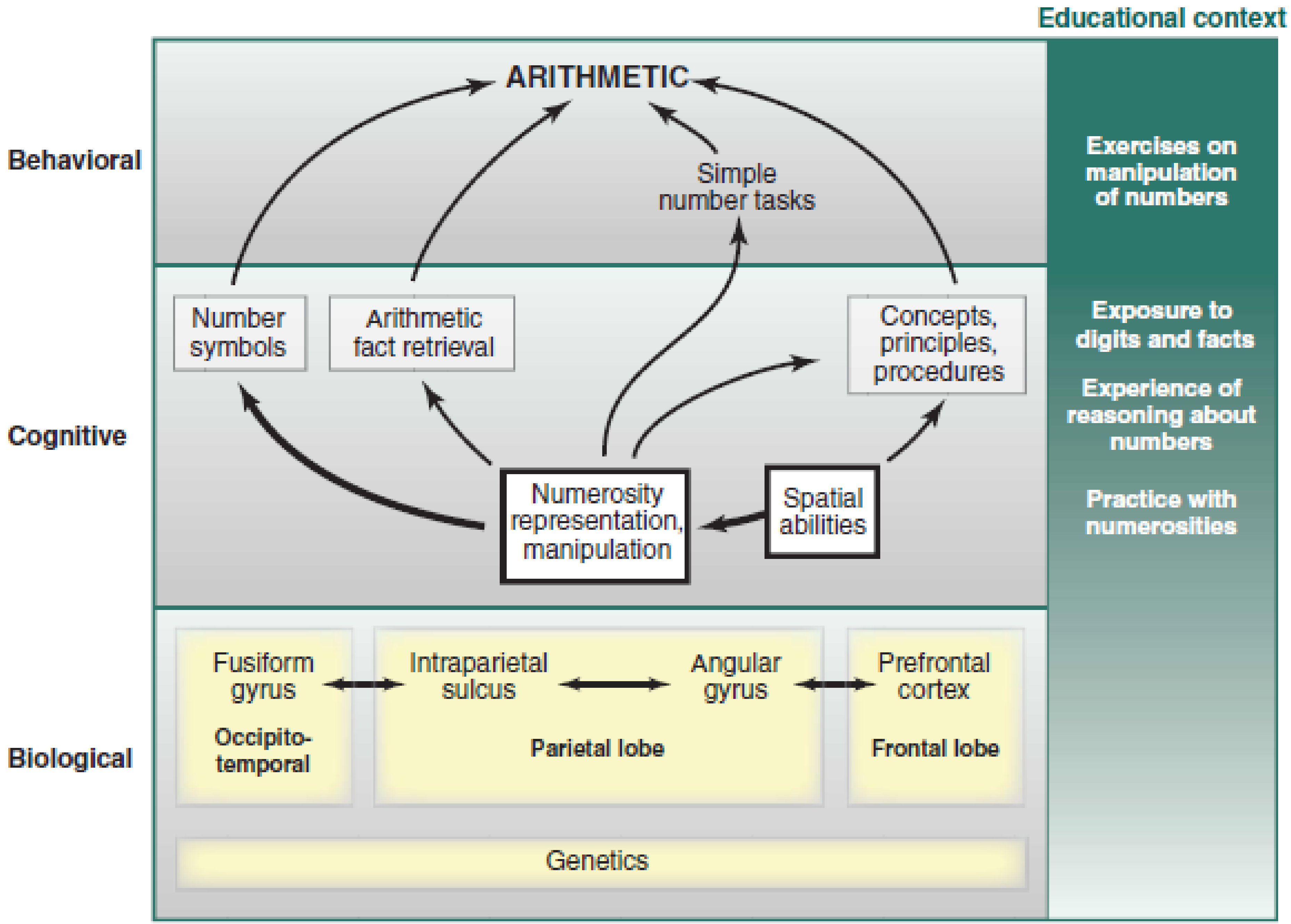


Fig. 1. Causal model of possible inter-relations between biological, cognitive, and simple behavioral levels. Here, the only environmental factors we address are educational. If parietal areas, especially the IPS, fail to develop normally, there will be an impairment at the cognitive level in numerosity representation and consequential impairments for other relevant cognitive systems revealed in behavioral abnormalities. The link between the occipitotemporal and parietal cortex is required for mapping number symbols (digits and number words) to numerosity representations. Prefrontal cortex supports learning new facts and procedures. The multiple levels of the theory suggest the instructional interventions on which educational scientists should focus.

Numerical development: Key issues

- Pre-verbal (non-symbolic) numerical abilities:
  - numerosity estimation, subitizing, addition/subtraction
- innate or acquired?
- what mental representation(s)?
- what relationship to mathematical learning?

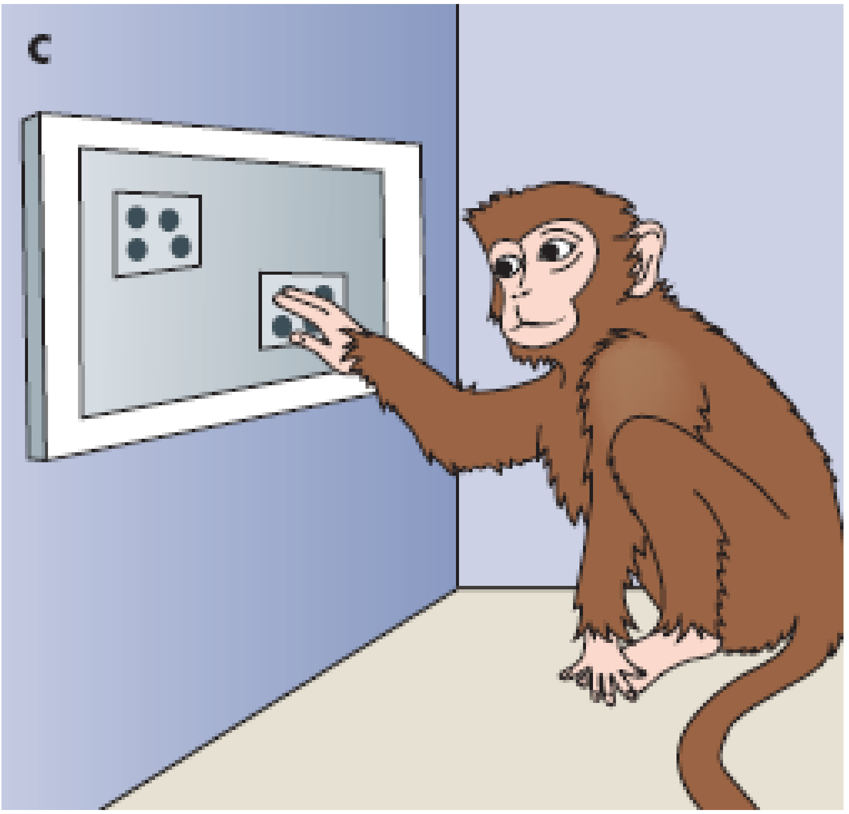
The “number sense”

1. The ability to attend to numerosity and to manipulate it internally (*number sense*) is present in a variety of animal species, even without training

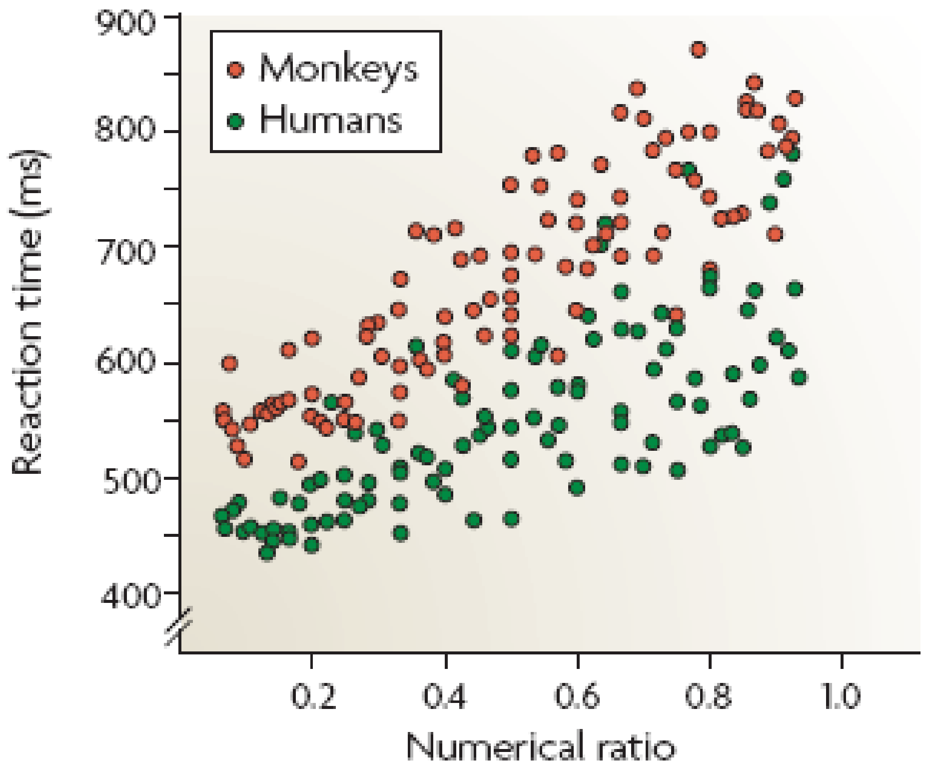
2. A similar capacity for elementary number processing is found early on in human development, prior to schooling and to the development of language skills

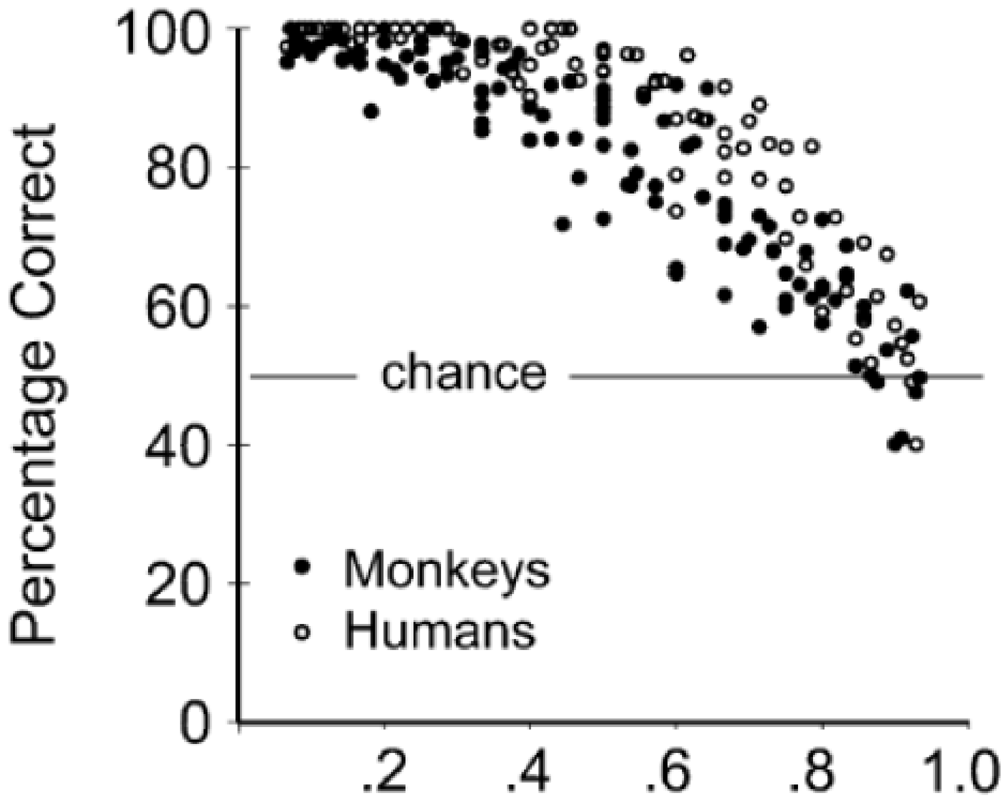
3. Number processing relies upon specific neural circuits – and their disfunction is related to number processing and calculation deficits (dyscalculia)

Numerosity comparison



The similarity in the performance of monkeys and humans suggests a phylogenetic continuity in the representation of numerical magnitude.
(Cantlon & Brannon, 2006, *Psych. Science*)



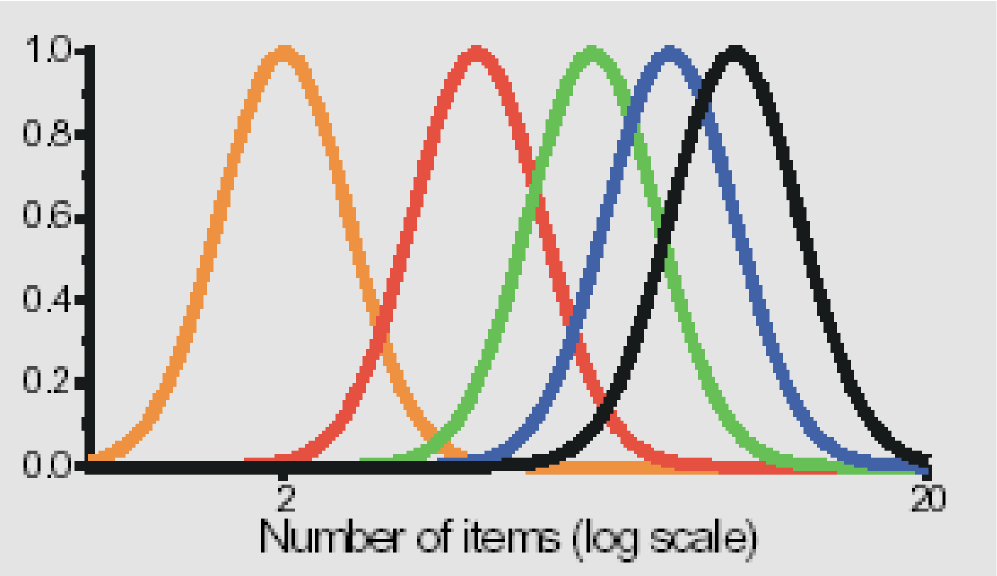


A Fechnerian encoding of numerical magnitude?

The representation of numerical magnitude is compressive (i.e., it obeys Weber-Fechner logarithmic law).

(Dehaene, 1992, 2003)

Thus, the subjective difference between two numbers will depend on their positions on the number line, that is, the subjective difference between N and N+1 will be smaller as N increases.

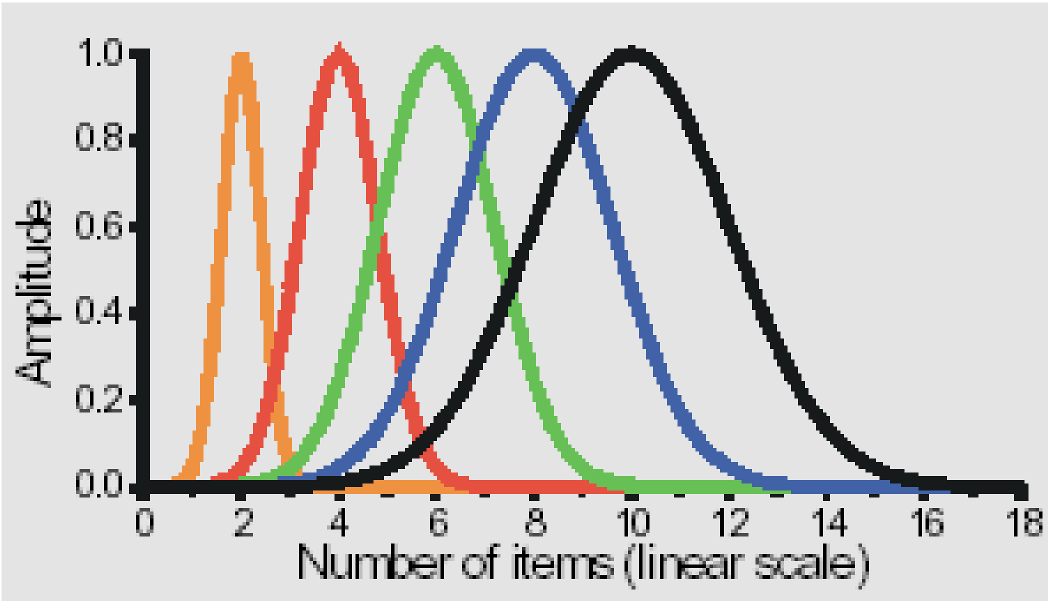


Or a "noisy" mapping to the number line?

The mapping from numerals to the number line is linear, but the variability of the mapping increases in proportion to the magnitude (scalar variability)

(Gallistel & Gelman, 1992, 2000)

Thus, the discriminability of the two numbers decreases as their mean numerical value increases, not because they are subjectively closer together, but because the variability (noise) in the mapping is scalar.



Numerical processing in Infants

Infants: from few hours to12 months

Limits: language, arousal, physical abilities, etc.

Specific paradigms:

- habituation
- violations of expectations

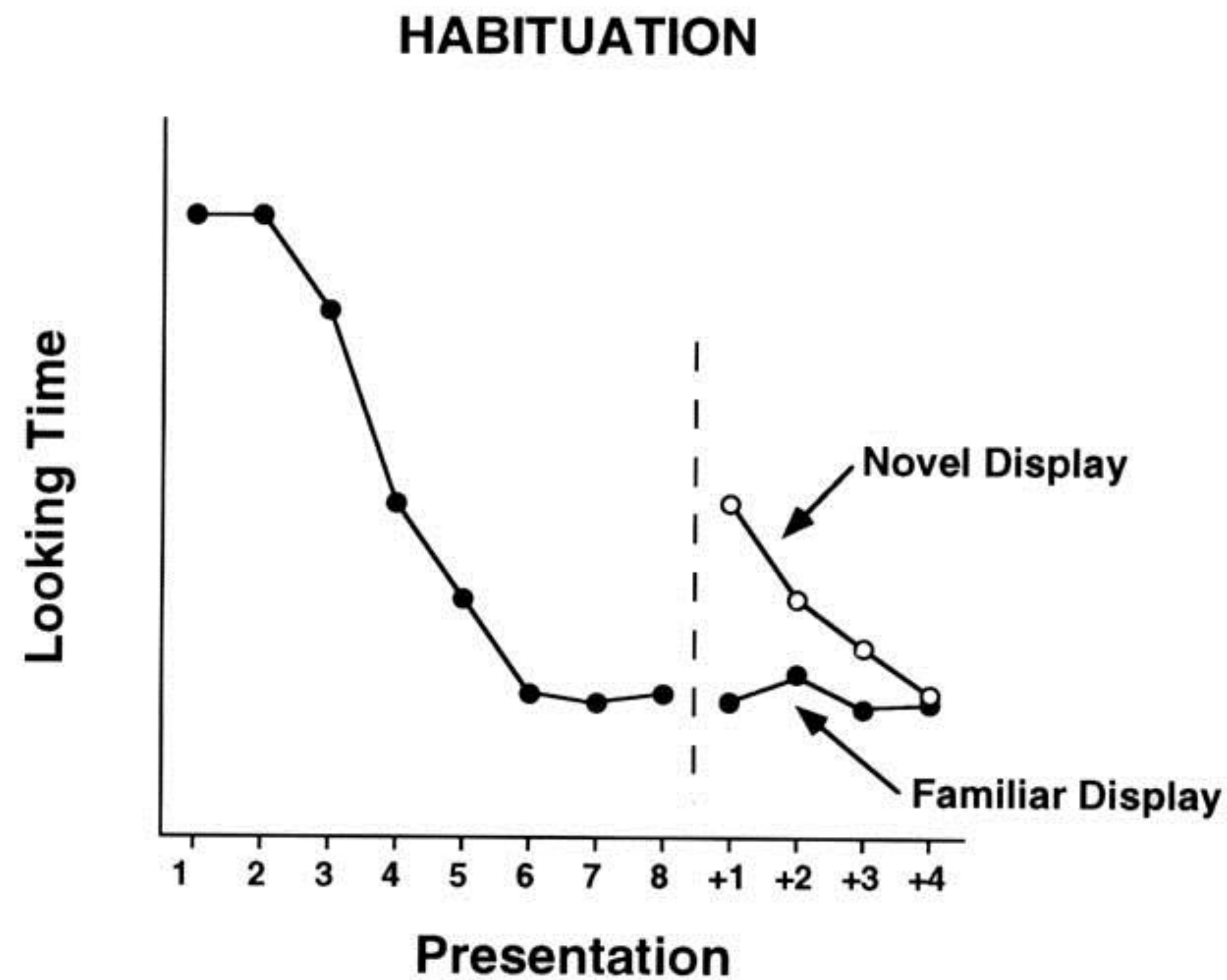
Habituation

Assumption: novelty increases attention/arousal

eye gaze = attention/arousal

2 phases:

- habituation
- test

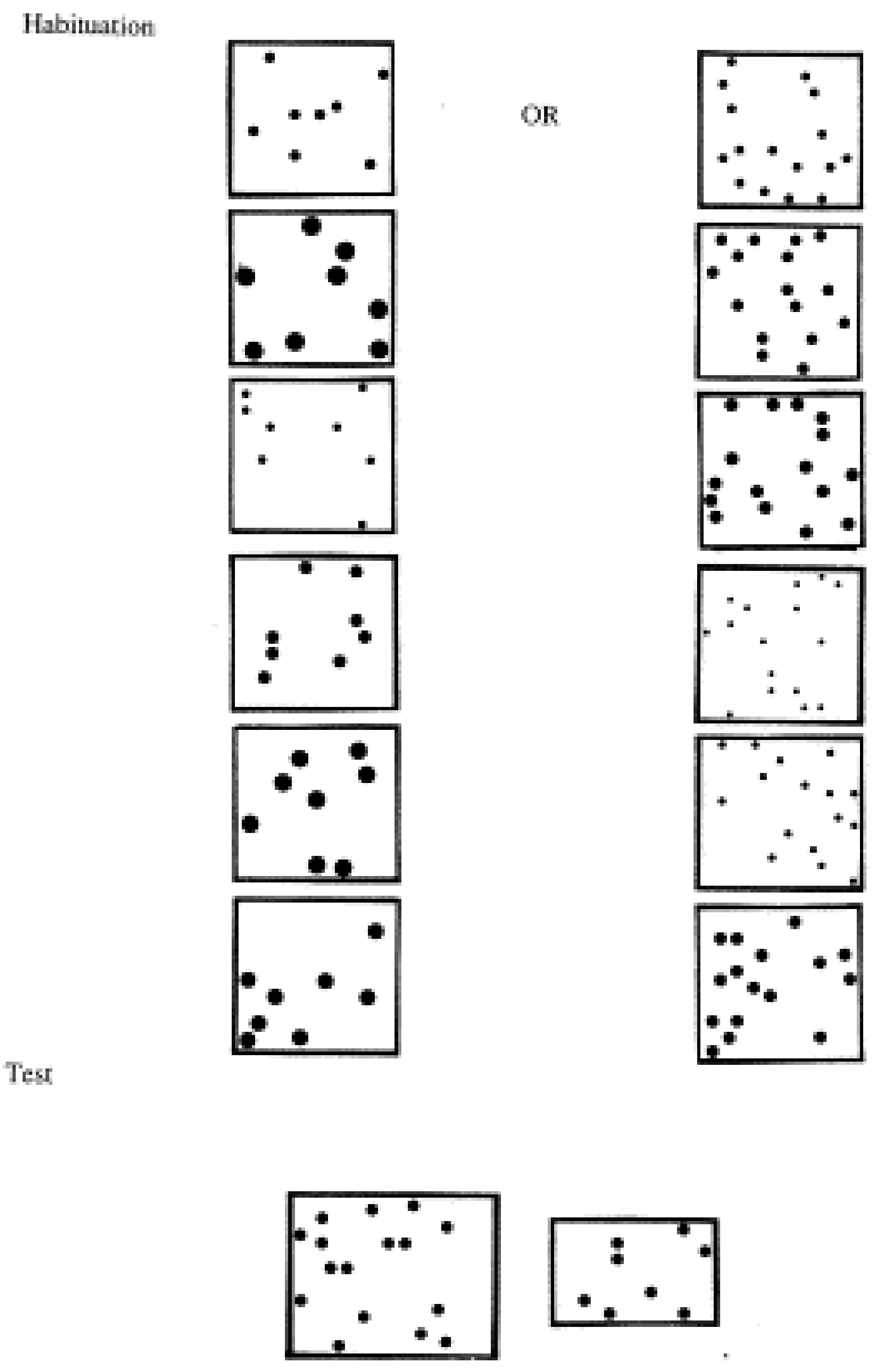


Large number discrimination: the ratio effect

- Newborns discriminate between sets if ration is 1:3, e.g. 6 vs. 18 dots (Izard et al., 2009, *PNAS*)

- 6-months-old discriminate between sets if ratio is 1:2. e.g. 8 vs 16 dots (Xu & Spelke, 2000, Cognition)

9-months-old discriminate between sets if ratio is 2:3, e.g. 8 vs 12 dots (Lipton & Spelke, 2003, Cognition).



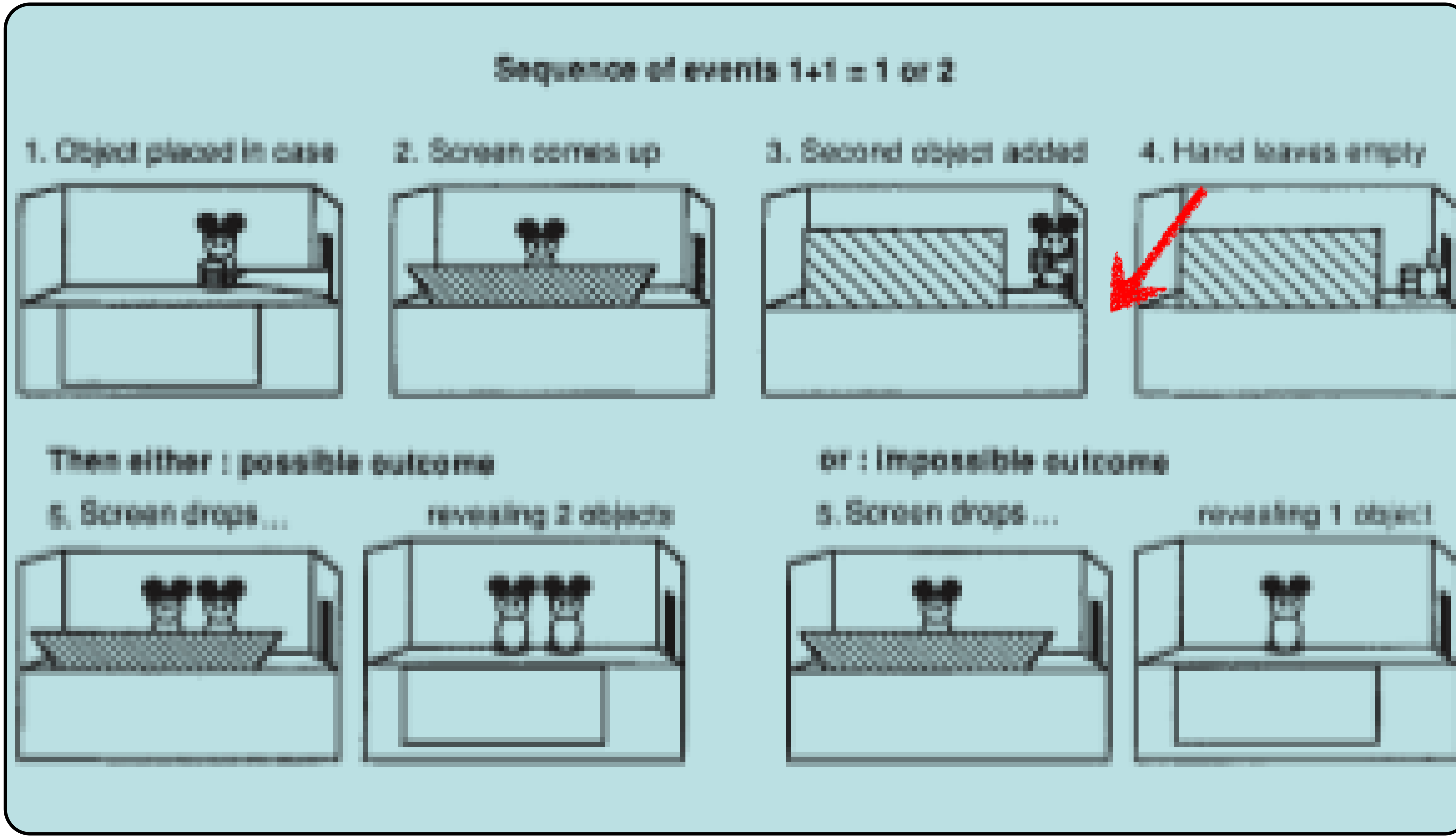
Violations of Expectations

Assumption: infants are sensitive to a wide range of physical properties of objects

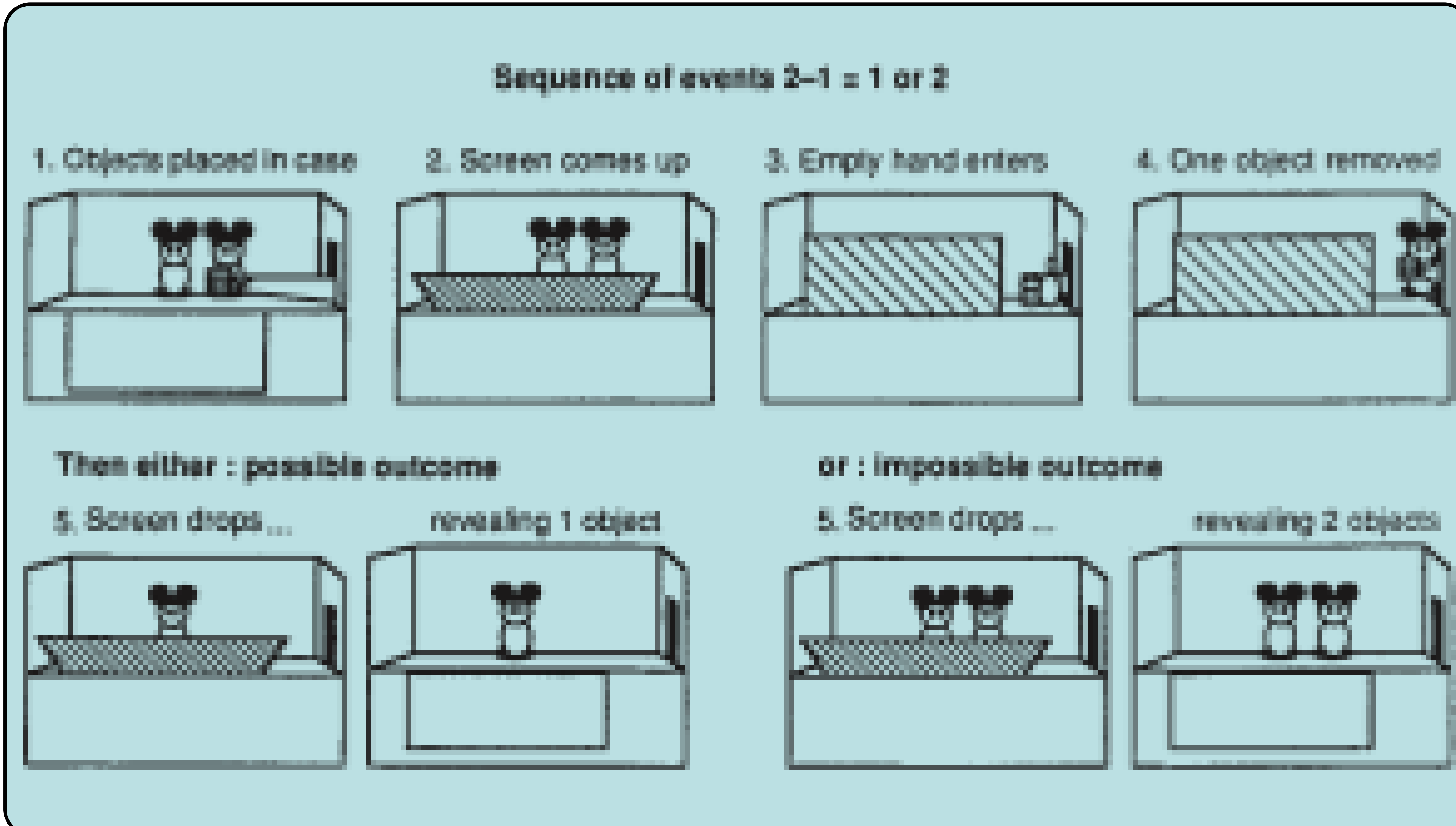
"naive physics": permanence, volume, solidity, spatio-temporal continuity (Spelke, 1988, 1994, Spelke et al., 1992)

If violation of expectation-> increase of attention = longer looking time

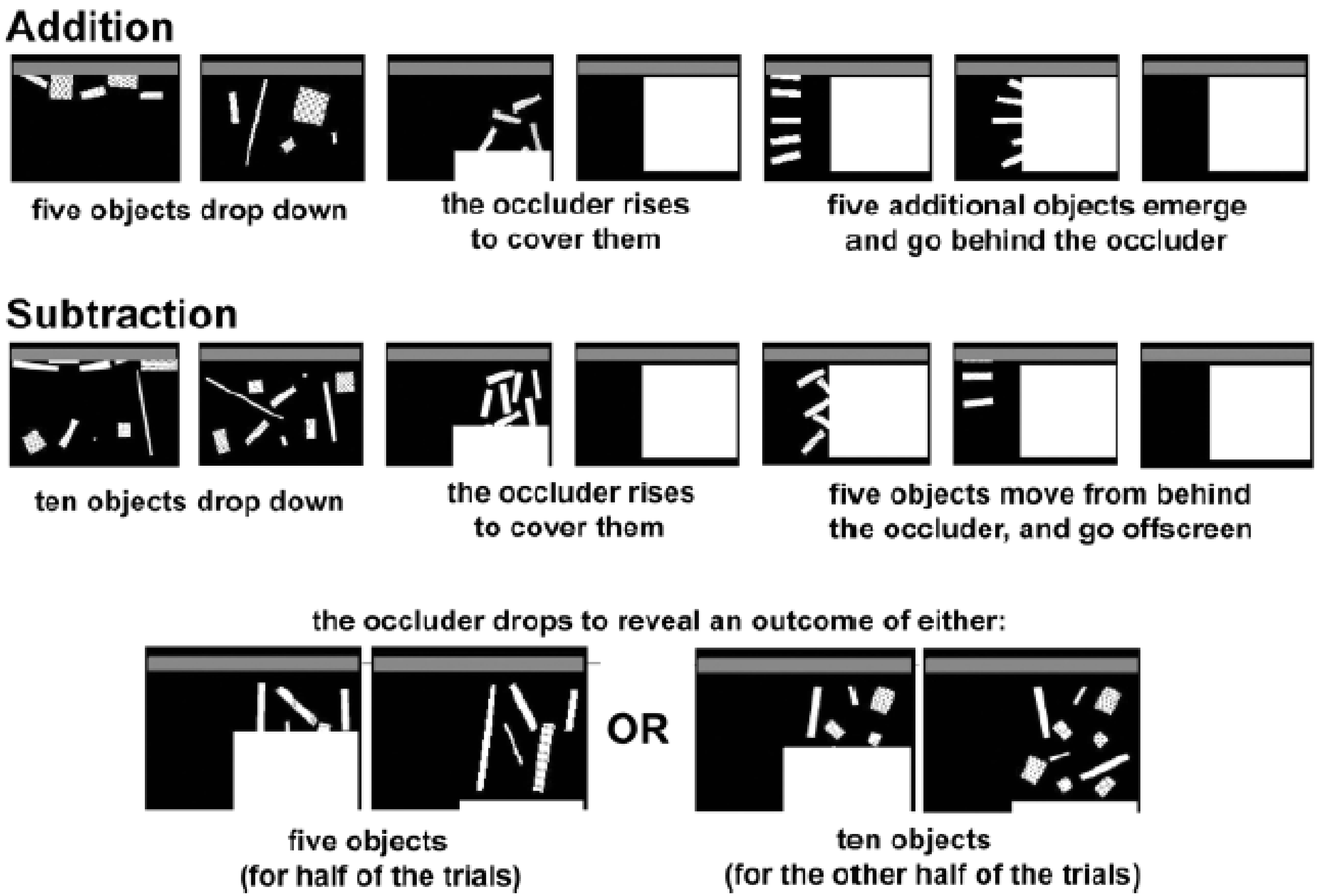
Addition and subtraction by infants



(5-month olds; Winn, 1992, *Nature*


Sequence of events 2–1 = 1 or 2
1. Objects placed in case
2. Screen comes up
3. Empty hand enters
4. One object removed
Then either : possible outcome
5. Screen drops ...
revealing 1 object
or : impossible outcome
5. Screen drops ...
revealing 2 objects

Approximate arithmetics: large numbers



9-month olds; McCrink & Wynn, 2004, *Psychol. Science*

Interim summary

Humans are endowed from the earliest days of the ability to discriminate numerosity

Importance of the ratio between sets (for numerosities > 4)

Innate expectations of addition and subtraction