5

Mathematics Disorder

In comparison to some of the disorders dealt with so far, particularly disorders of reading and language, there has been less research into the nature and causes of mathematical difficulties in children. Interest in these disorders seems to be expanding rapidly now. In many ways there are clear parallels between work on children's mathematical difficulties and research on children's reading difficulties. In both cases, we have an example of a specific difficulty in acquiring a critical educational skill. It seems likely that one of the reasons why less research has been devoted to children's mathematical difficulties than children's reading difficulties is that the typical developmental pattern for mathematical skills is more complex, and harder to understand, than in the case of reading. This in turn makes attempts to understand mathematical difficulties all the harder.

We will refer to problems in this area as mathematics disorder (MD). Alternative terms from adult neurology, particularly dyscalculia (literally an impairment of calculation) and acalculia (literally an inability to perform calculation), are still sometimes used to refer to mathematics disorder in children.

Definitions and Prevalence

The Diagnostic and Statistical Manual of Mental Disorders (DSM-IV; American Psychiatric Association, 1994) defines mathematics disorder as follows "mathematical ability, as measured by individually administered standardized tests is substantially below that expected given the person's chronological age, measured intelligence and age-appropriate education." This definition is essentially identical to the equivalent definition of reading disorder quoted in Chapter 2. This is an explicitly developmental definition, since mathematical skills need to be below the level expected for a person's age, intelligence, and educational experience. In contrast to studies of reading disorders, there has not been a great controversy about the use of a discrepancy definition of mathematics disorder (a definition that requires mathematical abilities to be out of line with intelligence). Some of the same issues raised in relation

to a discrepancy definition of reading disorder apply to mathematics disorder as well (see Chapter 2). While there is no doubt that on average children of higher IQ tend to be better at arithmetic than children of lower IQ, this does not demonstrate that IQ is a cause of arithmetic problems, and there are rare cases of individuals of very low IQ who have superior calculation abilities (e.g., Hermelin & O'Connor, 1991).

The prevalence of mathematics disorder is less well documented than for reading disorders. In the UK, Lewis, Hitch, and Walker (1994) assessed reading, maths (arithmetic), and nonverbal ability using group tests in all 9- and 10-year-old children in a single school district. There were some problems with ceiling effects on the tests (which may lead to underestimates of the number of children with difficulties). They used a simple cut-off approach to defining difficulties (rather than a regression-based approach). A child was regarded as showing a specific difficulty if their standard score on the maths achievement test was less than 85 (roughly the bottom 15% of the population) in the presence of normal nonverbal ability and reading (at or above a standard score of 90 on both). Using this criterion, they reported that only 1.3% of children showed a specific impairment in maths but another 2.3% had both maths and reading problems defined in an analogous way. Thus, 3.6% of these children would be classified as having a mathematics disorder. Another large-scale study using laxer criteria (a standard score of 92, the bottom 30% of the population) found combined rates of mathematics disorder (with and without reading problems) of 11.2% (Share, Moffitt, & Silva, 1988). A large US study reported a figure of around 6% (Baker & Cantwell 1985) and in Israel Gross-Tsur, Manor, & Shalev (1996) reported a prevalence of 6.5%. Both Lewis et al. and Gross-Tsur et al. showed that it is common for mathematics disorder to co-occur with reading disorders. Given the different populations studied and the different criteria for classification adopted it is difficult to compare the prevalence rates from these different studies. It is clear, however, that mathematics disorder occurs quite frequently and that these problems are often associated with reading disorders.

As in studies of dyslexia, studies of mathematics disorder have usually used children selected according to an IQ discrepancy definition. The aim of using such a definition is to identify children with mathematical difficulties that cannot be explained in terms of more general learning difficulties. In some studies to be considered below, a direct comparison has been made between children with mathematics disorder and children with a mathematics disorder combined with a reading disorder (mathematics disorder/reading disorder).

The Typical Development of Number Skills: A Theoretical Framework

In order to consider the problems some children experience in mastering mathematical skills we first need to consider how such skills typically develop. Laying out the typical course for the development of numerical and mathematical skills is difficult, because the skills involved are complex and quite diverse. How, for example, are counting, arithmetical calculations such as addition and multiplication, and higher

$2 + 4$ $18 - 6$ $54.01 + 48.89$ 45×5

$\frac{3}{4} - \frac{1}{4} =$ $\frac{2}{5} = \_\_\_\_ \%$ $13\overline{)403}$ $\frac{1}{6} + \frac{1}{3} =$

What is 15% of 180?

A car's fuel consumption is 10 **miles** per liter.
How many liters will it use over a 94 **kilometer** journey?
__________ liters.

Figure 5.1 Examples of the types of questions used in a standardized arithmetic test. (Adapted from the British Ability Scales II; Elliott, 1996.)

mathematical skills such as geometry and algebra related to each other? In what follows we only consider the development of basic number skills (understanding what numbers represent, and in turn the ability to count) and the development of basic arithmetic operations (particularly addition and subtraction). Most of the research on mathematics disorder has focused on children identified as having problems in learning basic arithmetical skills as measured by standardized tests. For example, the Wechsler Objective Number Dimensions test (Rust, 1996), the British Ability Scales (Elliott, Smith, & McCullouch, 1978) and British Ability Scales II (Elliott 1996) include tests of written arithmetic that begin with simple addition and subtraction problems suitable for young children and progress to more complex sums including multidigit multiplication and long division (see Figure 5.1).

Number concepts and the development of counting

Preverbal numerical abilities

In the last decade or so great excitement has been generated by the discovery that some primitive numerical abilities are possessed by animals and preverbal human infants (for a review see Dehaene, 1997). For example, rats can be trained to press one lever when two light flashes or two tones occur and another lever when four light flashes or four tones occur. When subsequently the animals are presented with mixtures of events that are synchronized (a flash and a tone at the same time) they respond in line with the number of events (the number of lights and tones; Church & Meck, 1984). This suggests some basic appreciation of numerical quantity in these animals. Perhaps more impressively, Woodruff and Premack (1981) trained a chimpanzee to select a physically matching stimulus – to choose a half-full glass of liquid that matched another rather than to choose a glass that was three-quarters full. What would the chimp now do if they were shown a half-full glass, and had to choose whether to match it with half an apple or three-quarters of an apple?

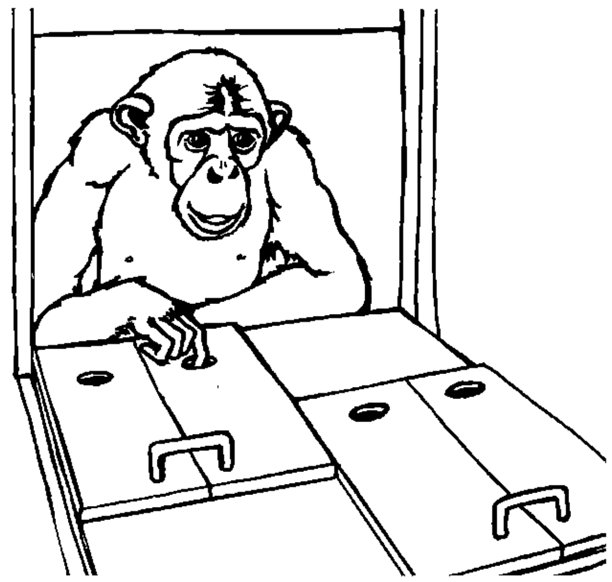


Figure 5.2 A chimpanzee in the study by Rumbaugh et al. (1987) selects a tray containing the larger number of food items. (Rumbaugh, D., Savage-Rumbaugh, S., and Hegel, M., Summation in the chimpanzee (Pan troglodytes), *Journal of Experimental Psychology: Animal Behavior Processes*, 13(2), 109, 1987, published by the American Psychological Association and reprinted with permission.)

The chimp chose the half apple, but clearly this cannot be based on any simple perceptual attribute.

Chimps will also select a tray with the larger number of food objects on it, without training, even when the food objects are arranged in a way that is misleading if they could not perform something akin to basic addition (Rumbaugh, Savage-Rumbaugh, & Hegel, 1987; see Figure 5.2). For example, a chimp might be offered a tray with a pile of five chocolate pieces and a pile with just one piece (six in total), versus a tray with a pile of four and a pile of three pieces (seven in total). The chimp will choose the tray with seven pieces, but this shows they must appreciate that the two small piles are greater than a large pile and a pile with just one chocolate in it. Such abilities at first seem very surprising, but arguably there has been considerable evolutionary pressure for animals to deal with numerical quantities and relationships when foraging for food.

Similar evidence suggests that human infants have some basic preverbal understanding of number. In a striking demonstration of this, Wynn (1992) showed 5-month-old infants a toy, which was then covered by a screen, then another identical toy was shown being placed behind the screen. When the screen was removed there were either one or two toys present; infants showed surprise (looked longer) when only one toy was present when the screen was removed. It seems the babies were expecting two objects because they had seen an object added to the place where there was already one object. With larger numbers of objects, when care is taken to control for cues such as surface area and density, it seems that 6-month-old infants

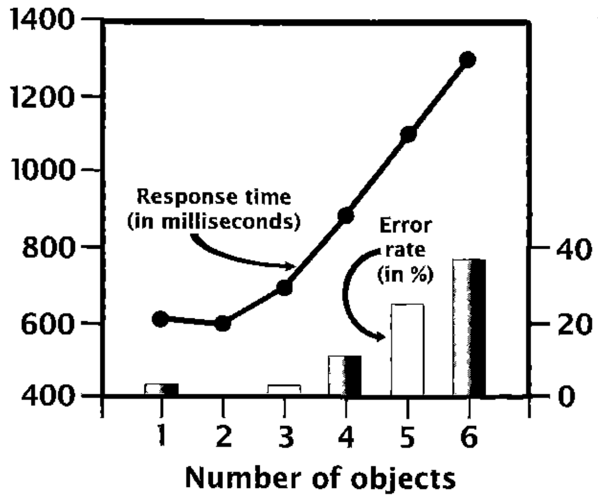


Figure 5.3 Subitizing: Reaction time is fast in identifying between one and three items but increases steeply thereafter. (Mandler, G. and Shebo, B. J., Subitizing: An analysis of its component processes, *Journal of Experimental Psychology: General*, 111 (1), 1–22, 1982, published by the American Psychological Association and adapted with permission.)

can discriminate between groups of 8 and 16, or 16 and 32 dots, but not between 16 and 24 dots (Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005).

Evidence such as this from studies of animals and preverbal human infants suggests that some basic numerical skills exist in the absence of language. This numerical system is probably somewhat imprecise and can only deal with small numbers of objects. Nevertheless, it has been suggested that such a preverbal "number sense" may form a foundation for more complex verbally elaborated number skills in humans (Dehaene, 1997). Evidence from older children and adults for such a rapid, if approximate, number processing mechanism comes from "subitizing" (Mandler & Shebo, 1982; see Figure 5.3). If people are shown displays of randomly arranged dots they can say how many are present equally rapidly for displays of between one and three; however for displays of four or more there is a clear increase in the time taken to respond as the number increases. This suggests that people can directly apprehend (grasp) differences in the number of objects present up to three but that after this a slower and more effortful mechanism akin to counting must be employed.

Finally there is evidence from a variety of sources that our understanding of numbers depends upon continuing access to some form of preverbal magnitude-based system. One sort of evidence comes from studies involving judging the relative magnitude of different numbers. In this task (Moyer & Landauer, 1967) people are simply presented with pairs of digits (e.g. 3 vs. 4 or 2 vs. 8) and asked to decide as quickly as possible which digit represents the large magnitude by pressing a key. The finding is that people are quicker to make such judgments when the difference between the digits is larger (people are quicker to choose 8 as the larger digit when it is paired with 2 than when it is paired with 7). This finding is referred to as the symbolic distance effect (SDE; see Figure 5.4); such an effect is the opposite to what

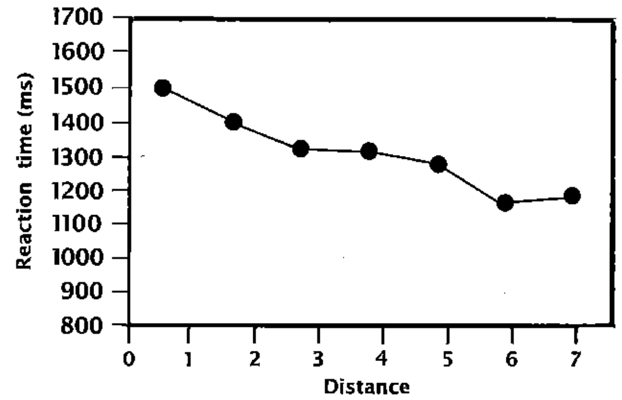


Figure 5.4 The time taken by 6-year-old typically developing children to judge which of two simultaneously presented digits represents the larger quantity (for digits 2–9). Digit pairs that are similar in numerical magnitude (a symbolic distance of 1: e.g., 2 vs. 3; 3 vs. 4; 4 vs. 5; etc.) are hard to make judgments about (slow reaction time); as the symbolic distance between pairs of digits increases, the task becomes progressively easier (faster reaction times).

we would expect if comparing the numbers involved counting. Instead the pattern parallels a pattern found when we compare the physical size of objects and provides evidence that even in adulthood our understanding of the magnitudes represented by numbers depends on accessing some form of analogue representation.

Such findings have been related to the idea that the magnitudes represented by numbers may depend upon access to a mental number line, where numbers are arranged in positions from left (small) to right (large; see Figure 5.5). This is an idea put forward a long time ago by Galton (1880). A further characteristic of the mental number line is that it seems to involve some sort of unequal spacing (or compression) as we go from small to large numbers, that is, the distance on the number line between 1 and 2 is larger than the distance between say 8 and 9. Evidence for this idea of a nonlinear number line comes from the finding that in the Moyer and Landauer task people are quicker to judge that 2 is larger than 1 than they are to judge that 9 is larger than 8 (the problem size effect). Note, once again, that if such comparisons were based on counting these two problems should be equally easy. As we shall see later there is now some evidence that children with mathematics disorder suffer problems in the nonverbal representation of numerical magnitudes.

The development of counting

Gelman and Gallistel (1978) studied children from around 2 years upwards and proposed that learning to count depended on a number of "how to count" principles:

1 The one-to-one principle: Each object to be counted gets one and only one count word.
2 The stable order principle: The count words (one, two, three, four ...) must be used in a fixed order.

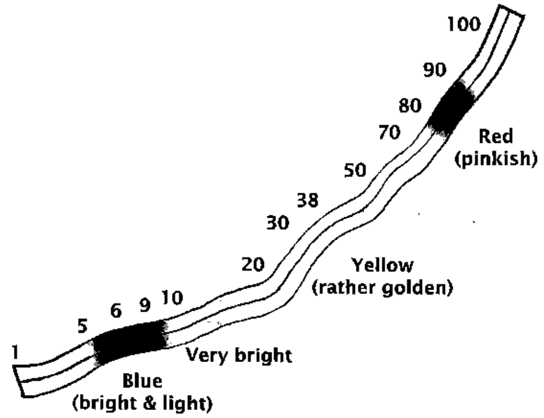


Figure 5.5 A diagram of the mental number line. After Galton, 1880, who suggested that people imagined colors at different points on the number line. (Galton, F., Statistics of mental imagery, *Mind, 19*, 1880, pp. 301–318, by permission of Oxford University Press.)

3 The cardinality principle: The last count word used represents the cardinal value, or number of things in the set, that has been counted.

Gelman and Gallistel also proposed a further two principles that they considered less fundamental:

4 The abstraction principle: Any collection of objects can be counted (how many pieces of fruit are here?).
5 The order irrelevance principle: The order in which objects are counted has no effect on the outcome.

In Gelman and Gallistel's view these early "how to count" principles depend upon basic innate constraints on development that guide the development of effective counting skills. In this view, the principles outlined above are somehow known before children have learned to count. Such a claim is controversial and more recent evidence suggests that understanding the "how to count" principles may emerge from prolonged practice of using counting procedures (Rittle-Johnson & Siegler, 1998; Sophian, 1998). By whatever means children master counting, it is clear that it provides a critical foundation for the more advanced arithmetic operations such as addition and subtraction that are taught in school. Counting is fundamentally a form of measurement and one that is more flexible and precise than the form of measurement revealed in subitizing or in studies of animals' and infants' preverbal numerical abilities. Just how (or even if) preverbal number skills feed in to the development of verbal counting skills in older children remains an important, if unresolved, issue. Recent evidence (Gilmore, McCarthy, & Spelke, 2007) suggests that some approximation skills come quite naturally to young children when presented with simple addition and number comparison tasks. However, such ease with approximation tasks contrasts with the marked

difficulty many children have in mastering basic computation skills, perhaps suggesting a disjunction between primitive estimation skills and exact computation skills.

The Typical Development of Early Arithmetic Skills

By the time children go to school they are generally proficient at counting, at least for numbers up to 10, and these counting skills form a foundation for the development of arithmetic skills. To become numerate demands more; children need to learn conventional systems and to use their mathematical thinking meaningfully and in logical situations (Nunes & Bryant, 1996).

We will concentrate on the developmental pattern found in studies of pre-school and primary-school-age children (mostly in the USA and the UK). Children are expected to master a range of arithmetical skills during the primary school years. These skills will involve (roughly in the following order) single digit addition, subtraction, multiplication, and division, and the extension of these skills to multidigit numbers. Children in the later primary school years are also required to master fractions and deal with proportions and percentages.

Single digit addition

One of the complexities in studying arithmetical development is that often a given problem can be solved in different ways (using different procedures or strategies). There is a great deal of evidence showing that the strategies children use develop in sophistication with age and practice. This is well illustrated by studies of simple addition.

Understanding how to add a pair of single digits (e.g., 2 + 2) is the first type of arithmetic operation to be formally taught in school. Even single digit addition is a complex skill. Young children, before they have entered school, may first learn to perform addition using the sum procedure. So, given the sum 2 + 3, a young child might count aloud or on fingers "one, two ... three, four, five" – the last count word used here corresponds to the sum, just as in counting a set of objects. However a more effective counting strategy for solving such a sum is a count on strategy whereby the child states the number represented by the first digit and then counts up from there "two – three, four, five." Finally, the most sophisticated strategy involves the realization that identifying the larger of the two digits and then counting up from that involves less counting (such a strategy should therefore be easier and less error prone). This is referred to as the min strategy and involves the child understanding the commutativity principle: changing the order of the numbers in the sum does not alter the result (2 + 3 is the same as 3 + 2).

There is evidence that understanding commutativity is related to children using the min procedure for addition. For example, Baroody and Gannon (1984) assessed commutativity by showing kindergarten children pairs of sums (e.g., 2 + 4 and 4 + 2) and asking them to judge whether the answer to both sums would be the same. Evidence for commutativity involved children answering such questions quickly and without counting. Children who did this were more likely to solve simple addition sums (e.g., 2 + 6) using the min strategy. However, it was not uncommon for children to show understanding of commutativity but not to use the min strategy, which suggests that children may first need to understand commutativity before they can go on to apply this to selecting the min strategy when faced with an addition problem (Baroody & Gannon, 1984; Cowan & Renton, 1996).

This discussion of the very early stages of learning addition brings out a fundamental distinction that is central to understanding the development of arithmetic. This is the distinction between conceptual and procedural knowledge (essentially the distinction between knowing and doing). When a child is given a problem to solve, they may select a count all or a count on procedure (with or without consistently applying a min strategy). These two procedures are both "correct" in that they will give the right answer. However, even in this very simple case the child may "know" something (commutativity) that does not necessarily translate directly into what they "do" (using a min strategy to solve the problem).

Thus, addition, which is the earliest arithmetical skill to be taught in school, can be seen as a natural extension of counting. Later, as children learn the number bonds, they can begin to retrieve these automatically. Development involves change in the mix of strategies that are used. Importantly, the development in long-term memory of an association between the problem integers (e.g., 3 + 4) and the answer that is generated (7) requires practice in the execution of basic computations. With each execution, the probability of direct retrieval of that number fact or bond increases. This direct retrieval strategy is rapid and highly efficient and is the culmination of many less automatic computations of the relevant sums. (An analogy might be drawn with basic reading skills where after some limited number of effortful decodings of a word the child learns to retrieve the correct pronunciation and associated meaning of a printed word relatively rapidly and effortlessly.) It follows that children who have difficulty with the more basic count-based strategies for addition will take a long time to acquire a database of number facts, and they may therefore fail to achieve automaticity in arithmetic skills (Geary, 1993). Furthermore if count-based addition procedures are inaccurate and error prone this may lead to considerable problems if incorrect problem solutions are stored in permanent (long-term) memory.

How are number bonds stored in memory?

A number of formal models have been proposed for how the knowledge underlying the direct retrieval of answers to addition problems is stored in long-term memory. Ashcraft (1982, 1987, 1992) proposed a model in which arithmetic facts are stored in an associative network with retrieval occurring via a process of spreading activation. In this model there is a two-dimensional table, with addends (the digits to be added) along each side of the table, and the answer is obtained from combining the two addends at the intersection (activating the row for "2" and the column for "3" activates a cell corresponding to 5). The more frequently the input nodes corresponding to two addends are simultaneously activated along with the correct answer (activating the input nodes 3 and 2 along with the output node 5), the more accurately and quickly the correct answer will be retrieved. Activation in this network spreads more quickly for smaller valued problems (Ashcraft & Battaglia, 1978). This assumption of the model provides an explanation for a fundamental aspect of arithmetic performance, referred to as the problem size effect: People, even adults, are typically faster and more accurate when adding smaller (3 + 2) than larger pairs of digits (8 + 9).

Campbell (1995) proposed a more complex model of both addition and multiplication fact retrieval in which there are physical codes for digits (the symbols 1–9) that are associated with magnitude representations (an analogue magnitude representation of the sort proposed by Dehaene, 1997). The magnitude representations are less precisely specified for larger magnitudes and this accounts for the greater difficulty in distinguishing between pairs of large magnitude numbers than small magnitude numbers (Dehaene, 1997; Moyer & Landauer, 1967). In terms of this model, retrieving the answer to a given addition problem (2 + 3) depends upon associations between representations in the physical code (associations between the symbols 2 and 3) as in Ashcraft's model, as well as associations between these codes and the magnitude representations they represent. The lesser precision of the magnitude representation for larger problems in this model provides an explanation for the problem size effect (slower and more error-prone performance for large problems). Furthermore, it is postulated that the number of potentially interfering associations between physical codes tends to increase as digit pairs get larger. Thus in this model as problem size increases the number of competing associations in the physical code representations increases and the precision of the associated magnitude code decreases, and these influences both tend towards slower and more error-prone responses to larger problems.

Siegler's model (Siegler, 1988; Siegler & Shrager 1984) is simpler in that associations between digit pairs and both correct and incorrect answers are stored in memory according to the frequency with which the digit pair has been associated with the different answers. The stronger and more numerous the associations between a sum and incorrect solutions in memory, the slower and more error prone will be retrieval of the correct answer. In this view the problem size effect should be strong in children (where many inaccurate solutions may be stored with appreciable frequency) but this effect should disappear with extensive practice (the associations between 3 + 2 → 5 and 8 + 9 → 17 should eventually be equally easy to retrieve if each problem has been encountered and answered correctly enough times). In this model, the problem size effect is essentially a form of frequency effect, such that problems that are encountered often yield faster and more accurate responses.

The details of these different models are not crucial for present purposes. All the models agree that associations between number representations are formed in memory, and these associations are influenced by the frequency with which they occur. All of the models considered would anticipate that errors in counting during problem solving by children will tend to make their retrieval of the correct answers to addition problems slower and more error prone, and that is the basic pattern found in children with mathematics disorder. Much more research would be needed to identify whether detailed patterns of addition performance in children with mathematics disorder can be used to constrain or test these different models.

Summary of the typical development of arithmetic

We have concentrated on the development of addition skills for pairs of single digit numbers because these have been so thoroughly studied and some of the principles revealed probably apply generally to other aspects of arithmetic (subtraction, multiplication, and division). Even in what is the simplest example of arithmetic (adding two digits) there are multiple procedures (strategies) that a child might use. The evidence is that children typically move through these strategies in the order described (count all, count on, count on from min) initially using fingers as an aid to counting. It is important to note that all three strategies are correct in the sense of yielding the correct answer, but the developmental sequence is one of moving from a less efficient, more effortful, procedure to a more efficient one. Finally such computational strategies gradually generate knowledge in long-term memory with practice and repeated use so that when given a sum (e.g., 3 + 5) the child can rapidly retrieve from long-term memory the correct answer (8). This is an example of the cumulative nature of mathematical development. If a child has problems in counting, these will lead to problems in executing basic procedures for addition, which in turn will lead to problems in creating an effective knowledge base of number facts in long-term memory. This raises the possibility that mathematical difficulties observed in older children (say addition problems in a 9-year-old child) may depend upon difficulties with basic procedures at an earlier stage (problems in learning to count accurately when the child was much younger).

The Nature of Arithmetic Difficulties in Children with Mathematics Disorder

Before considering the possible cognitive causes of mathematics disorder, it is useful to describe the pattern of difficulties shown by these children on basic arithmetic tasks. Geary (1990) studied a group of 29 children with "learning disabilities" (LD) who had weak reading and arithmetic skills with an average age of 8 years 4 months. These children were reassessed on standardized tests 10 months after their initial tests and 16 children with continuing problems in arithmetic were identified. Each child completed a set of single digit addition problems presented on a computer screen, which allowed the speed of the child's spoken response to be recorded. Observations made on each trial noted whether the child counted aloud or on their fingers, or used their fingers in another way. The 16 children with continuing maths difficulties were more likely to make errors in counting when solving these simple addition problems, and though they were less likely to retrieve answers from memory for the problems, on the trials when they did so they were more likely to make an error. Thus these children's addition was inaccurate compared to that of children of their own age when using either a counting or a direct retrieval strategy. The LD children when using counting strategies did not count more slowly than the typically developing children, though their speed of counting was more variable. Overall, the

pattern reported by Geary is compatible with the idea that the children with LD (who had both mathematics disorder and reading disorder) have problems in counting accurately. Such problems with counting might account for difficulties in solving addition problems with a counting strategy. In addition the frequent counting errors made by these children may also contribute to the difficulties they have in using a direct retrieval strategy (because the counting errors they make when solving addition problems tend to lead them to store faulty representations of number bonds in long-term memory). However, one problem in interpreting the difficulties shown by this group of children is that the majority appeared to have impairments of both reading and arithmetic.

Jordan, Hanich, and Kaplan (2003) conducted a 16-month longitudinal study of four groups of children: mathematics disorder (MD), reading disorder (RD), mathematics disorder/reading disorder (MD/RD), and typically achieving children matched for age. There were just over 40 children in each group who were selected by giving group tests of reading and arithmetic to over 600 7- to 9-year-old children. The MD and MD/RD children did not have particularly severe arithmetic difficulties, with an average arithmetic percentile score of 22 (MD) or 21 (MD/RD) (i.e., on average these children were in the bottom 22% of children in terms of their achievements on the standardized arithmetic test used).

The children were given a battery of arithmetic measures on four occasions: place value, requiring the child to identify which digit in a two- or three-digit written number corresponded to the number of units, tens, or hundreds; calculation principles, in which the child had to respond quickly to the second of two sums, where the second sum could be solved easily on the basis of having given the answer to the first (e.g., 47 + 86 = 133; so 86 + 47 = ?); number fact retrieval, involving a speeded measure of simple addition; exact calculation, involving a set of eight written addition and subtraction problems; story problems, where the child was presented orally with an arithmetic word problem to solve; and approximate arithmetic, where children had to select the answer to a sum that was closest to the correct value (e.g., 4 + 5 = 10 or 20?). The results of this study were clear in showing that the MD/RD children tended to have more severe difficulties on all of the arithmetic tasks than the MD, group (and significantly more severe problems on exact calculation, story problems, and calculation principles) even though they had not differed significantly in terms of the standardized test on which they had been identified at screening. Perhaps surprisingly, both the MD and MD/RD children showed the same rates of improvement on arithmetic measures as the control group. The MD/RD group tended to use an immature addition strategy (finger counting) more than did the RD and control groups.

This study shows that the problems with arithmetic identified in the 7–9-year-old range tend to be stable, and surprisingly there were no differences in the rate of development of arithmetic skills in the MD group compared to control children (however, arguably these children did not have severe difficulties to begin with). The MD children tended to use immature, slow, and error-prone calculation strategies and to have problems in retrieving correct solutions from long-term memory. One further important finding is that the MD/RD children clearly had more severe problems than the MD children. Finally, the RD children in this study showed a tendency to have weaker arithmetic skills than controls on most measures (except approximate arithmetic). This finding suggests that the phonological deficits found in the RD children may have some small effects on arithmetic, but these problems are different to those causing difficulties for the MD children. In this view the MD/RD children may suffer from two relatively independent deficits: a phonological deficit (perhaps giving rise to problems learning and executing the count sequence) and a more basic arithmetical deficit (which is not phonologically based but is present also in the MD children).

Cognitive Bases of Difficulties in Children with Mathematics Disorder

Studies of children with mathematics disorder (with or without a reading disorder) have investigated a number of the components of arithmetic development outlined above. Likely potential causes of arithmetical difficulties include:

1 Number (magnitude) representation problems: basic difficulties in representing numbers (learning number symbols (5) and number words (five) and mapping these onto the underlying magnitudes they represent).
2 Counting problems.
3 Number fact storage problems: difficulties in learning and storing the solutions to problems (e.g., 3 + 5 = 8) that form the basis of the direct retrieval strategy for mental arithmetic problems.
4 Attentional control and working memory problems: problems in executing the processes (strategies) required to solve a problem because of problems in storing and manipulating information in working memory or problems of attentional control involved in selecting and monitoring the execution of these strategies.

We have listed these potential problems in order of their complexity and we will consider each of them in turn. This is not an exhaustive list of possible cognitive deficits and nor are the deficits mutually exclusive (different children might have some, all, or none of these deficits). It is also worth noting that these potential cognitive deficits might, developmentally, be causally related to each other. For example, initial problems in representing numbers might in turn lead to problems in counting, which in turn may lead to problems in learning problem solutions that are to be stored in memory. Such problems with counting and/or retrieval processes will in turn place extra demands on attentional resources and might lead to apparent working memory/attentional control difficulties. We are arguing here that there may be a developmental "cascade" of difficulties, with problems with elementary processes early in development (e.g., counting problems) leading to other problems later in development (e.g., problems with direct number fact retrieval). However, depending upon the age at which children are studied, the original problems may have largely resolved in groups of older children (an 11-year-old child with mathematics disorder may appear to count competently, but that does not mean that problems with counting when they were 5 years old may not have contributed to the problems now observed). These possible interrelationships between difficulties makes identifying the basic, or primary, cognitive causes of mathematics disorder particularly challenging.

Problems of number representation

There is a limited amount of evidence that children with mathematics disorder have basic problems with number representation. Geary, Hoard, and Hamson (1999) found that a minority of children with mathematics disorder/reading disorder could not name "12" when it was presented visually, and that some of these children could not write "13" when it was dictated to them. These children were accurate, however, when given equivalent tasks with single digit numbers. Furthermore they report that for a group of MD children (who were of higher IQ than the MD/RD group) there were no equivalent problems in reading or writing these numbers. Unfortunately only a small array of numbers was assessed in this study, there were no measures of speed taken, and accuracy levels were essentially at ceiling. A small number of the MD and MD/RD children in this study were also reported to make errors on an untimed digit comparison task where they were required to choose the digit representing the larger number from a pair (e.g., 5–7) while age-matched typically developing children were essentially perfect on this task. Landerl, Bevan, and Butterworth (2004) studied 10 RD, 10 MD and 10 MD/RD children who were around 8–9 years old. Unfortunately, standard scores for these groups' reading and arithmetic skills are not given, and nor was any information presented on their verbal IQ (groups were matched on nonverbal IQ), making their cognitive profiles difficult to discern. It appears that the RD group had age-appropriate arithmetic skills and severely deficient reading skills, the MD group had weak reading skills and moderately impaired arithmetic skills, and the MD/RD group had severe reading problems and similar arithmetic skills to the MD group. The children were asked to name single and double digit numbers and color patches presented on a computer screen. The results suggested that only the MD group were slower than controls to name single digit numbers, while all three groups (MD, MD/RD, and RD) were slower than controls to name double digit numbers. The RD group were slowest to name the colors but the MD and MD/RD groups also appeared somewhat slow on this task.

These children were also asked to make speeded comparisons of the magnitude of single digit numbers (which is the larger number: 4 vs. 6?) or the physical size of two numbers (which is bigger: 4 vs. 6?, when the physical size of the digit varied independently of its numerical magnitude). There were no differences between the four groups of children on the physical size judgment task, which rules out any general differences in perceptual or motor speed between the groups that could affect their judgments of numerical magnitude. However, the MD and MD/RD groups were slower than both the control and RD group in judging numerical magnitudes. This is an important result and provides support for the idea that children with mathematics disorder (with or without accompanying reading disorder) have a basic deficit in representing numerical information. Similar effects were obtained by Passolunghi and Siegel (2004), who compared a group of 22 10-year-old children with mathematics disorder to a group of 27 control children matched for age and vocabulary knowledge. (It should be noted that although the MD and control children did not differ on a standardized measure of reading comprehension, there was a moderate difference between the groups on this measure ($d = 0.38$) and no assessment of reading accuracy or speed was taken – hence these children may not be a totally pure MD group). The MD children were slower and less accurate in making odd/even judgments about single and multidigit numbers, and slower in making magnitude judgments about 16 single and multidigit numbers.

Finally, in a larger-scale study, Rousselle and Noël (2007) examined the speed of digit magnitude judgments (which is the larger number, 2 or 4?) in 42 children with mathematics disorder (16 of whom had mathematics disorder/reading disorder) and 42 age-matched control children (the children were 7 years old, and those with mathematics disorder were in the lowest 15% of the population for mathematical skills). The MD and MD/RD children showed equivalent levels of performance on this task, and were much slower than controls (effect size $d = 1.2$). Strikingly, these same groups did not differ in their ability to make speeded judgments about the number of lines presented in two sets side by side on the computer screen (which group of lines contains more?). Hence the difficulty in children with mathematics disorder appeared specific to accessing numerical magnitudes, and did not extend to judging numerosity. The evidence from these group studies is paralleled by an earlier case study of an adult university student with dyscalculia. Butterworth (1999) described the case of "Charles" who showed severe deficits on a number comparison task.

In summary, the studies of Landerl et al. (2004), Passolunghi and Siegel (2004), and Rousselle and Noël (2007) provide support for the claim that MD children may have a very basic deficit in representing the meaning of numbers (the magnitude signified by a digit). This is a deficit not shared with RD children. The finding of these low-level problems in representing numerical magnitudes in MD children suggests that some aspect of a preverbal "number module" or "number sense" (as proposed by Dehaene, 1992; Butterworth, 1995) may be a core feature of children with mathematics disorder. It seems possible, but far from certain, that this problem may in turn contribute to problems in learning to count.

Counting problems in children with mathematics disorder

Application and understanding of count principles

Children with mathematics disorder have problems in learning to count. Geary, Bow-Thomas, and Yao (1992) studied counting in a group of 13 7-year-old children with mathematics disorder/reading disorder. These children scored between the 2nd and 42nd percentile on an arithmetic test (meaning there was a wide range from severe to mild arithmetical difficulties in the group). It is mentioned that many of these children had associated reading problems, though no information on the children's reading skills, or IQs, is presented. In this study the children watched a puppet count an array of objects and had to indicate whether the puppet had counted correctly or not. Sometimes the puppet counted correctly but on other trials made an error by violating one of Gelman and Gallistel's principles of counting (such as counting either the first or last item in the array twice – a violation of the one-to-one correspondence principle). These MD/RD children, compared to control children of the same age, often incorrectly accepted trials when the first object was counted twice as correct. It was suggested that this might have reflected a difficulty holding the information about the initial count in memory until the children were allowed to respond to say whether the count was correct or incorrect. The MD/RD children were also more prone than control children to wrongly indicate that trials on which the puppet did not count adjacent items consecutively were wrong. This suggests a limited understanding of the essential features of counting, though one might argue that the children here were sensitive to the fact that counting things in a nonadjacent order is a nonoptimal strategy that might easily lead to errors in counting.

In an extension to this study (Geary et al., 1999) children in Grades 1 and 2 of at least low-average IQ with either mathematics disorder, reading disorder or both (mathematics disorder/reading disorder) were selected. The results essentially replicated those of Geary et al. (1992) in showing that both groups with mathematics disorder (MD and MD/RD) differed from children with reading disorder and controls on the nonadjacent count trials (wrongly claiming that these were incorrect) and on the first item double-count trials (for the younger children only). The MD children performed just like the MD/RD children, indicating that the earlier results of Geary et al. (1992) were unlikely to be due to including children with both mathematics disorder and reading disorder.

It appears that many young children with mathematics disorder have some limitations in their understanding of the conceptual basis of counting, though the problems they have in this domain do not appear to be severe. Geary et al. (1992) argued that the MD children in their study largely understood Gelman and Gallistel's three fundamental principles of how to count (one-to-one invariance, stable order, and cardinality), though they tended to see some irrelevant aspects of counting (adjacency) as important, just as younger typically developing children often do (Briars & Siegler, 1984).

Problems with counting speed

Passolunghi and Siegel (2004) found that their group of 10-year-old children with mathematics disorder were slower to count arrays of 7–10 dots on cards than were controls. Landerl et al. (2004) asked their children with mathematics disorder, mathematics disorder/reading disorder and reading disorder to count as quickly as possible from 1 to 20, from 45 to 65, and from 1 to 20 in twos. Both the MD and MD/RD groups were slower at counting, particularly in the higher range of numbers tested, and when counting in twos. The RD group were also somewhat slower but not as slow as the MD and MD/RD groups. These findings must be treated as tentative given the small sample sizes involved, but they suggest that children with mathematics disorder may be slower to learn to count, and that when they are older they remain slow at counting.

It appears from these studies that children with mathematics disorder may have a basic deficit in number representation (as assessed by their difficulties on the number magnitude judgment task described earlier). In contrast, children with reading disorder do not have such a deficit, but do share with MD children some difficulty in counting (less severe than the problems encountered by MD children). Tentatively, we suggest that both of these problems (number representation and counting difficulties) might contribute to the problems of learning arithmetic experienced by MD children.

Problems in storing numerical information in long-term memory

Children with mathematics disorder seem to have great difficulty retrieving number facts from memory. There are at least two ways of thinking about this problem. First, it might simply reflect the fact that these children have not had the typical opportunities to learn and store this information in long-term memory. Given that these children make frequent counting errors when trying to solve addition problems, it is possible that they have limited opportunities to learn the correct answer to a problem because they generate so many incorrect solutions (which may also be stored in memory and so contribute to slow and error-prone performance). This idea probably provides a partial reason for the problems these children have in retrieving number facts from memory but it seems unlikely that it provides a sufficient explanation for most of the problems observed.

A second possibility is that there are problems in either encoding information into memory or in storing it adequately once it is encoded. In this view even when a child with mathematics disorder correctly generates the answer to a problem by counting (3 + 5 = 8; correct!), this information either does not get encoded into the long–term memory system or it is not stored efficiently (some models of how such number fact storage may operate were discussed earlier). The notion of an encoding or storage deficit of this sort is hard to test and there does not appear to be any direct evidence to support it.

This idea relates to questions of how such number facts are stored. Are the number fact storage mechanisms based on a common verbally based memory mechanism or are they dependent upon a separate system (a separable number fact memory system)? Brain damage in adults can produce highly selective deficits in arithmetic in the absence of deficits in spoken and written language processing. Such evidence certainly suggests that, at least in the adult system, the retrieval of number facts depends on a relatively independent memory system. Many patients have been described who show selective impairments of different aspects of number fact knowledge (addition, subtraction, multiplication; van Harskamp & Cipolotti, 2005), which further suggests separable storage systems for different aspects of number fact knowledge. Furthermore, aphasic patients have been described (Whalen, McCloskey, Lindemann, & Bouton, 2002) who are unable to generate a phonological representation of a number problem (they cannot read aloud the numbers correctly, nor perform judgments about phonological forms of the number words: Do 4 and *sour* rhyme? Do 4 and *pour* rhyme?) but nevertheless can retrieve number facts reasonably accurately. This suggests that storage of number facts in memory depends at least in part on a nonphonological code. This might mean that number facts are stored in an abstract meaning-based code (McCloskey & Macaruso, 1995), in some very abstract speech-based code (Dehaene & Cohen, 1995), or in multiple (phonological and semantic) codes, as in Campbell's multicode model described earlier.

It seems possible that children with mathematics disorder do suffer from a specific deficit in the long-term storage of number facts in memory (though convincing evidence for this idea still needs to be found). The evidence from neuropsychology certainly indicates that in adults the storage of number facts depends upon one or more relatively abstract codes that are independent of other phonological or semantic memory representations. It is plausible, therefore, that children with mathematics disorder might experience a specific problem in establishing such memory representations, perhaps wholly or partially, as a consequence of the more basic problems in representing numerical magnitudes described earlier. In Campbell's (1995) model of addition for example, number fact retrieval depends upon the activation of a magnitude representation, and such representations appear to be impaired in children with mathematics disorder.

Working memory problems

The term working memory (WM) refers to the ability to store and process information at the same time (Daneman & Carpenter, 1980; Just & Carpenter, 1992). Arithmetic is one of the clearest examples of a "real life" working memory task. Consider being presented with the problem "14 plus 17" in spoken form. To answer this problem you have to remember the two numbers (addends), retrieve and execute the appropriate procedures, and finally articulate the answer. This involves holding information in memory while at the same time retrieving and operating on other information.

Hitch (1978) provided a classic demonstration of the role of working memory in arithmetic. Hitch asked adults to solve orally presented multidigit addition sums (e.g., 423 + 63). The most frequent calculation procedure used by adults here would be to add the units first, then the tens, and finally the hundreds. Hitch varied a number of aspects of the task to manipulate the load imposed on working memory. On some trials people could write down the answer in right-to-left order (starting with the units and so lessening the load on memory) while on other trials they had to write the answer in left-to-right order (so the entire sum had to be solved before any of the answer could be written down). Errors increased when the answer had to be written in the order imposing the higher memory load (left-to-right). Errors decreased when part of the sum (the first, second, or both addends) was presented in written form to reduce memory load and errors increased when the number of "carry" operations increased. All these results are consistent with the idea that working memory storage demands are one source of difficulty in performing mental arithmetic.

Working memory, as used so far, is a theoretically neutral term in relation to the specific mental processes involved. Working memory storage depends upon multiple interacting systems with different coding and storage processing limitations. According to one influential model (Baddeley, 1986; Baddeley & Hitch, 1974) it is necessary to distinguish mechanisms specialized for the retention of visual information (the visuospatial sketch pad) from mechanisms specialized for the retention of

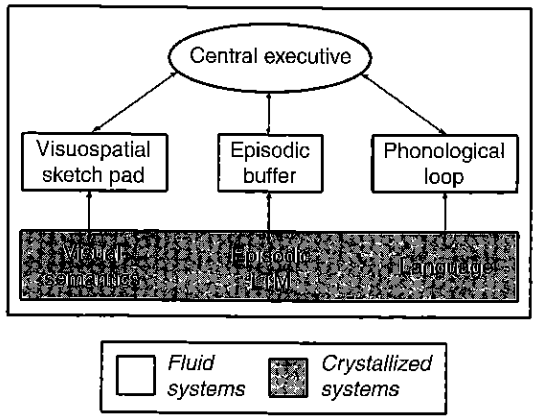


Figure 5.6 Diagram of Baddeley's working memory model. LTM = long-term memory. (Reprinted by permission from Macmillan Publishers Ltd, *Nature Reviews Neuroscience* (vol. 4, p. 835, Baddeley, A. D., Working memory: Looking back and looking forward), copyright 2003.)

phonological information (the phonological loop), and both of these systems interact with an attentional control system (the central executive). More recent versions of this model have postulated further components, including an episodic buffer (Baddeley, 2003b; see Figure 5.6).

For present purposes we simply need to stress that multiple systems will be involved in the storage and processing of arithmetic problems. At a minimum we need to postulate a role for phonological coding, visual spatial coding, and central attentional processes. Given a spoken word problem ("seventeen plus twenty-three") people will likely generate a phonological representation in an obligatory fashion, but may also often elect to generate a visuospatial representation of the problem (an image of the corresponding problem written down). Executive or attentional resources are likely to be involved in creating and maintaining such representations, and also in retrieving the information and procedures (from long-term memory) that are required to solve the problem. Even for solving a simple problem, the array of information that may need to be retrieved from permanent memory can be considerable, including deciding on the correct procedure to use, knowledge of count sequences (more important in younger, less skilled children), knowledge of rote-learned number facts (more important in older, more skilled children), and perhaps higher-level strategies such as checking on whether the answer generated is plausible and, if it is, checking on whether it is correct or not (perhaps by recalculating, in the same or a different way).

This discussion makes it clear that inefficiencies in certain component processes are likely to contribute to increases in memory load during calculation. So, for example, if number fact retrieval is inefficient this could mimic a working memory limitation. Developmentally, increases in the efficiency of working memory skills appear to be associated with changes in a number of other processes. In particular developmental increases in working memory performance with age appear to be highly correlated with increases in the speed with which elementary cognitive processes can be performed (Kail, 1991).

We can see that to propose that mathematics disorder arises from a "working memory" problem is no more than to propose a set of quite diverse possible deficits. It is useful to distinguish three different ideas about possible working memory limitations as potential causes of mathematics disorder.

Problems in phonological memory?

Arithmetic clearly depends in part on the use of information held in a phonological code, as in the use of verbal counting strategies. Problems in holding and manipulating information in a phonological code might contribute to problems in arithmetic. To assess this possibility we need to focus on studies that have selected children with mathematics disorder without reading disorder because it is very well established that children with reading disorder show problems on phonological memory tasks.

The results from studies of children with mathematics disorder compared to age-matched control children suggest that problems in phonological memory tasks (recalling, in order, lists of spoken words) are either small or absent. McLean and Hitch (1999) compared a small group of 12 9-year-old children with mathematics disorder (whose arithmetic scores fell in the bottom 25% of a large group of children who had been tested) with an age-matched group of children with normal arithmetic scores who were closely matched for reading ability. No measures of general ability (IQ) were obtained. There was a difference in digit span scores between the two groups (an effect size of $d = 0.95$) that certainly would have been significant on a larger sample. However, such a difference might well reflect uncontrolled differences in general ability (IQ) between the groups.

Passolunghi and Siegel (2001) studied a group of 9-year-old children with mathematics disorder and compared them with a group of children matched for age, vocabulary, and reading comprehension skills. There were no differences between these groups on a measure of memory span for words. Similarly, Passolunghi and Siegel (2004) studied a group of 10-year-old children with mathematics disorder and a group of children matched for age and vocabulary skills. The two groups did not differ on memory span for words or digits (the children with mathematics disorder were actually slightly better at recalling the lists of words). Similar results to this were also reported by Temple and Sherwood (2002), who compared a group of children with mathematics disorder (an arithmetic age 12 months below chronological age; 6/10 of these children had a chromosomal disorder called Turner's syndrome) with an age-matched control group matched for verbal IQ. There was no sign of any difference between groups on measures of memory span for digits, or for lists of one-, two-, or three-syllable words. Geary, Hoard, and Hamson (1999) also found no difference in digit span between 15 children with mathematics disorder (without reading problems) and a group of control children with typical arithmetic skills matched for age and IQ.

Overall it is clear that deficits in phonological memory are not typically found in children with mathematics disorder when care is taken to exclude possible effects of poor reading or differences in general ability (IQ).

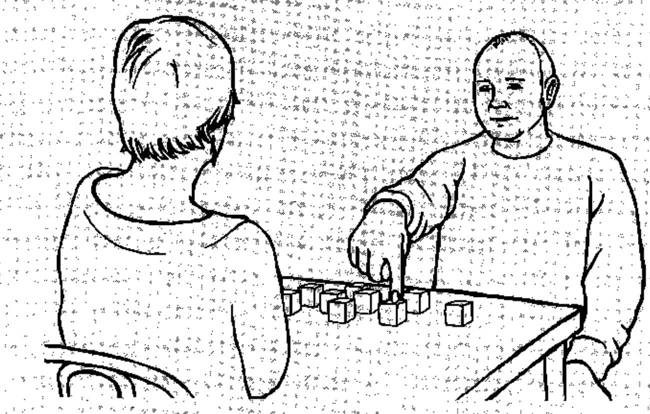
Problems in visuospatial memory processes

McLean and Hitch (1999) found large differences on the Corsi blocks test (imitating the order in which a number of blocks are tapped; Box 5.1) between children with mathematics disorder and age-matched controls, though such differences might reflect differences in general ability (IQ) that were not assessed. Temple and Sherwood (2002) reported no differences on this task between children with mathematics disorder and age-matched controls, however the difference between the two groups (a medium effect size, Cohen's d = 0.53) would have been significant on a larger sample. This effect, however, is in turn almost certainly associated with the lower spatial ability that is typically reported for children with Turner's syndrome. Finally, Bull, Johnston, and Roy (1999) found that 7-year-old children selected for high or low arithmetic ability did not differ (Cohen's d = 0.16) on Corsi blocks.

In summary, the few studies to date fail to find clear evidence of visuospatial memory problems in children with mathematics disorder, though the methodological limitations of the studies do not allow strong conclusions to be drawn. Since tests of spatial and nonverbal IQ typically correlate moderately with measures of arithmetic skill (Fennema & Sherman, 1977; McGee, 1979) it would actually be surprising

Box 5.1 Corsi blocks

The Corsi blocks task is a classic test of spatial memory. In this test, the examiner taps a series of blocks, starting with two blocks. The participant is required to reproduce the sequence by tapping blocks in the same order as in the demonstration. The examiner gradually increases the difficulty of the task by increasing the length of the sequence to be copied until a ceiling level (or spatial span) is reached. The task taps spatial memory and also places significant demands on executive skills.



Person taking the Corsi blocks test.

if children with mathematics disorder showed completely age-appropriate visuospatial memory skills.

Problems in attentional/executive processes

Typical measures of working memory (WM) involve the simultaneous processing and storage of information. For example in reading span (Daneman & Carpenter, 1980) people have to read a sequence of sentences aloud, answer a question about whether each sentence is true or not, and, after a sequence of sentences have been read, recall the final word from each sentence in the correct order (see Figure 3.6). In counting span (Case, Kurland, & Goldberg, 1982) the person is required to count the number of dots on a sequence of cards, and then recall the count totals in the correct order (see Figure 3.6).

One characteristic of these WM tasks is that they are very demanding of attention. Engle, Tuholski, Laughlin, and Conway (1999) gave a number of WM tasks, together with a number of conventional short-term memory (STM) tasks (digit span and word span) and tests of general fluid intelligence (GF) to a large group of adults. They showed that measures of WM were separate from (though correlated with) measures of STM. They argued that what was shared between the WM and STM measures reflected memory storage, and the "extra" thing measured by WM tasks was executive attention. When the variance common to WM and STM was statistically removed from the WM measures, these measures still correlated well with GF, but when the variance common to WM and STM was removed from the STM measures these measures no longer correlated with GF. Engle (2002) has argued persuasively that the WM construct is "related to, maybe isomorphic to, general fluid intelligence and executive attention" (p. 22).

There are now several studies of children with mathematics disorder showing deficits on complex WM tasks (such as listening span and counting span). For example, Passolunghi and Siegel (2001, 2004), in the studies described earlier, found that the children with mathematics disorder were worse on two WM tasks involving sentences as well as on a counting span task. As noted before, these same children did not differ on simple STM measures. On more classic "executive" tasks, Bull et al. (1999) found that their samples of 7-year-old children of high- or low-arithmetic ability differed on several aspects of the Wisconsin Card Sorting Test, notably perseveration (see also Bull & Scerif, 2001), and McLean and Hitch (1999) found that their mathematics disorder group did much worse than age-matched controls on a timed "Trail-Making" task. In this task, the children had to use a pencil to connect alternating sequences of numbers and letters (e.g., 1-A, 2-B) or numbers and colors (e.g., 1-yellow, 1-pink, 2-yellow, 2-pink). As noted before these groups were not equated for IQ, but the large differences between groups (Cohen's d = 1.5 on the colors trail-making task) coupled with the absence of appreciable differences on some other non-executive tasks suggest that these measures of executive function are probably not simply the product of uncontrolled differences in general ability between groups.

In summary, it is clear that there are large and consistent differences in executive function between children with mathematics disorder and typically developing children of the same age.

Summary of working memory in mathematics disorder

Children with mathematics disorder show deficits on complex working memory tasks, whether these tasks involve numbers (counting span) or not (listening span). In contrast, the same children in a number of studies do not show consistent differences on simple measures of verbal or visual short-term memory (recalling lists of words, or remembering the order in which a set of Corsi blocks is tapped). As outlined earlier, performing arithmetic clearly places heavy demands on working memory executive processes, and deficits revealed on such tasks are therefore likely potential causes of problems in learning arithmetic. However a number of caveats need to be considered.

First it might be argued that the working memory executive deficits found in children with Mathematics Disorder are so complex that they might always be reducible to some simpler underlying process. For example, Kail (1991) suggested that speed of processing may underlie increases in working memory capacity with age. It is at least plausible that the executive deficits found in children with mathematics disorder might be reduced to a processing speed deficit. Bull and Johnston (1997) found that processing speed accounted for unique variance in arithmetic ability among 7-year-old children after controlling for differences in reading ability; children with lower arithmetic ability were slower than controls at number naming and sequencing, number matching, pegboard speed, one-syllable speech rate, and reciting the alphabet. In contrast, the groups did not differ on short-term memory tasks. However, no IQ data were provided, and therefore processing speed may have been a proxy for general ability. Durand, Hulme, Larkin, and Snowling (2005) found that speed of information processing (as assessed by speed of visual search) was not a unique predictor of arithmetic skill after the effects of IQ and the speed of number comparison had been accounted for. This suggests that the speed of processing numerical information (rather than general information processing speed) may be critical for the development of arithmetic skills.

A second caveat concerns cause and effect relationships. A working memory executive impairment appears to be a highly general "nonmodular" deficit. As discussed earlier, Engle goes as far as to suggest that executive function "maybe isomorphic to general fluid intelligence." We need to be concerned therefore as to whether such a general deficit can really explain the highly selective deficits in arithmetic displayed by many children with mathematics disorder who, by definition, given the way in which they are selected, are often of normal IQ.

Variability among children with mathematics disorder

It may be that different children with mathematics disorder suffer from different underlying cognitive deficits (and, given the complexities involved in learning to do arithmetic outlined above, this seems quite likely). For example, some children might suffer from a deficit in counting, while others have a more fundamental problem with the representation of numerical magnitudes. In-depth single case studies of children with mathematics disorder have described different patterns of difficulty in different children. Macaruso and Buchman (1996) described a woman who had experienced problems in learning arithmetic throughout her life. She had great problems in number fact retrieval that did not appear to be associated with problems with counting, nor with any general difficulty in retrieving (non-numerical) information from long-term memory. This pattern suggests she had a specific problem in storing number facts in memory. In line with this, Badian (1983) suggested that some children with mathematics disorder have problems in learning and retrieving number facts while others have problems in dealing with the spatial layout of written arithmetic problems. O'Hare, Brown, and Aitken (1991) described the case of a child who had difficulty naming numbers or writing them to dictation, combined with other difficulties including problems in distinguishing left from right, in identifying their fingers, and in writing (a cluster of symptoms often referred to as developmental Gerstmann syndrome; Kinsbourne & Warrington, 1963; see Box 5.2). However, such difficulties in reading and writing numbers are rare among children with mathematics disorder, according to Badian (1983).

Geary (2004) has suggested that there may be three subtypes of mathematics disorder:

1. A procedural subtype in which children show problems learning to use simple arithmetical strategies that may be linked to verbal memory problems.
2. A semantic memory subtype associated with difficulties in retrieving number facts from long-term memory.
3. A visuospatial subtype involving problems with the spatial representation of number.

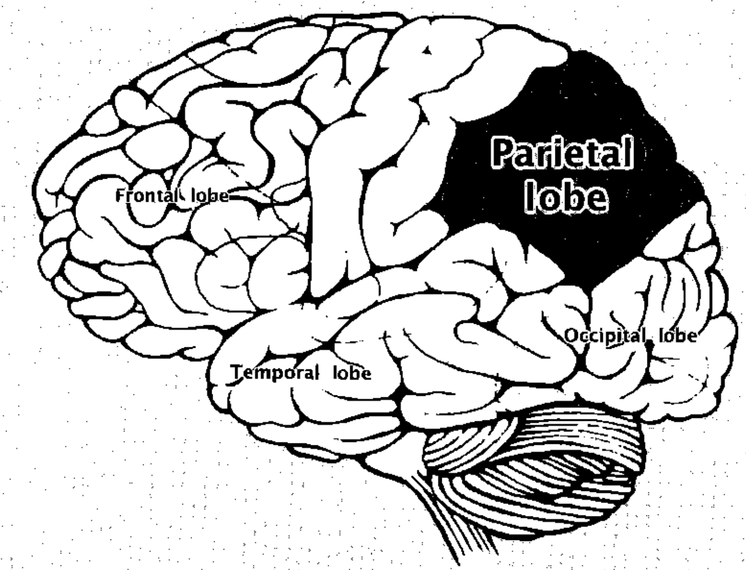
Box 5.2 Gerstmann syndrome (from *http://www.ninds.nih.gov/disorders/gerstmanns/gerstmanns.htm*)

Gerstmann syndrome is a neurological disorder characterized by four primary symptoms: a writing disability (agraphia or dysgraphia), a lack of understanding of the rules for calculation or arithmetic (acalculia or dyscalculia), an inability to distinguish right from left, and an inability to identify fingers (finger agnosia). In adults, the syndrome may occur after a stroke or in association with damage to the parietal lobe (see figure below).

There are reports of the syndrome, sometimes called developmental Gerstmann syndrome, in children. The cause is not known. Most cases are identified when children reach school age, a time when they are challenged with writing and math exercises. Generally, children with the disorder exhibit poor handwriting and spelling skills, and difficulty with arithmetic skills, including addition, subtraction, multiplication, and division. An inability to differentiate right from left and to discriminate among individual fingers may also be apparent. In addition to the four primary symptoms, many children also have reading

Box 5.2 *(cont'd)*

problems and difficulty copying simple drawings. Children with a high level of intellectual functioning as well as those with brain damage may be affected with the disorder.



Principal fissures and lobes of the cerebrum viewed laterally, highlighting the parietal lobe (adapted from the 20th US edition of *Grey's Anatomy of the Human Body*, originally published in 1918).

Related proposals were also made by Temple (1989, 1991), who described children with mathematics disorder who had problems with arithmetic procedures and those who had problems with number fact retrieval. Based on the review of earlier cognitive deficits we might add a "number sense" subtype with basic problems in understanding numerical magnitudes (Butterworth, 1999). These suggestions and the variations in the profile of difficulties shown by case studies of children with mathematics disorder are potentially important, and it would be useful for future studies to try to characterize the differences among children with mathematics disorder more carefully. One difficulty is that typically attempts to subtype children with mathematics disorder use descriptions that are closely related to details of their performance on simple arithmetic tasks. It will be more satisfactory, ultimately, if such subtypes could be based more clearly on underlying cognitive deficits that give rise to the differences in arithmetic performance that are observed. For the time being, however, there is too little evidence to say

that there are clearly distinguishable "subtypes" of mathematics disorder with different causes.

Summary of the cognitive bases of mathematics disorder

The nature of mathematics disorder is much less well understood than reading disorders, and this reflects the fact that we understand much less about the mechanisms of mathematical skills and their development than we do for reading skills and their development. For the most part in this chapter we have concentrated on the difficulties some children encounter when learning simple arithmetic skills (particularly learning addition). There is certainly much more to learning mathematics, or even to learning arithmetic, than learning about addition. However, the typical development of addition skills has been studied in more detail than other aspects of arithmetic, and children with mathematics disorder show very clear problems on this simple aspect of arithmetic.

Mathematical skills depend upon a complex interplay between nonverbal and verbal cognitive systems, and mathematical skills are arguably more diverse and more complex than reading skills. It seems likely from a cognitive perspective that mathematics disorder may result from a number of underlying deficits, including deficits in a nonverbal "number sense" system located in parietal brain areas, as well as verbal processes (such as counting) and executive processes that interact with this system.

It appears that further progress in this area is likely to depend upon longitudinal studies that attempt to focus on more well-defined arithmetical skills (such as counting, addition, subtraction, and multiplication). These studies should also seek to identify whether clear differences in the patterns of problems exist across children. At the moment, the paucity of longitudinal studies in this area is quite striking. Longitudinal studies are essential in order to describe how the profile of arithmetic problems changes with age. Such studies would allow us to assess whether particular cognitive deficits can be identified early in life that would reliably predict later problems in learning arithmetic. Such longitudinal predictive evidence will be critical for helping us to identify the cognitive causes of mathematics disorder.

The Etiology of Mathematics Disorder

Genetic influences on mathematics disorder

There is evidence for substantial genetic and environmental influences on the development of mathematical skills generally, and more specifically on the development of mathematics disorder. Conceptually, it is important to distinguish between genetic effects that operate to influence the development of normal variations in an ability (assessed by the heritability estimate for the ability) from genetic effects that operate to determine a disability (assessed by the heritability of group differences between people with a disability and people without it). It might be, for example, that particular genes influence whether individuals inherit a vulnerability to developing mathematics disorder. However, if these genes were uncommon in the population they might play no role in accounting for individual differences among people in the normal range of mathematics ability. In fact, the evidence suggests that the same genes that influence normal variations in mathematical skills in the population are also involved in influencing the development of mathematics disorder (Plomin & Kovas, 2005). We will consider briefly the evidence for genetic influences on both normal variations in mathematical skills and mathematics disorder. According to the arguments put forward by Plomin and Kovas (2005) these genetic influences are largely the same, which, if confirmed, would indicate that mathematics disorder is simply the lower end of the continuum of mathematical skill and not a discrete clinical entity.

There is good evidence that mathematics disorder tends to run in families (Shalev et al., 2001) but this may reflect either shared environment or genetic effects. Twin studies give one way of separating genetic from environmental effects. In the large-scale UK Twins Early Development Study (TEDS), Kovas, Harlaar, Petrill, and Plomin (2005) found evidence for substantial heritability for normal variations in mathematical skills in a sample of almost 3000 twin pairs. There were substantial overlaps between the genes responsible for arithmetic and general intelligence, and arithmetic and reading, though the degree of overlap was far from perfect, suggesting that there are specific genetic effects on the development of arithmetic skills.

There are very few studies that have directly assessed possible genetic influences on mathematics disorder (i.e., that have assessed the heritability of the group deficit in mathematical skills found in children with mathematics disorder compared to control children). Alarcon, DeFries, Light, and Pennington (1997) reported a study of the heritability of mathematics disorder in a small-scale study of 40 identical (MZ) and 23 nonidentical (DZ) twin pairs. At least one member of each twin pair had mathematics disorder (defined as a score on the standardized WRAT arithmetic subtest of 1.5 standard deviations below average, which corresponds to roughly the bottom 10% of the population). There were higher degrees of similarity in diagnostic category (mathematics disorder/control) for the MZ twin pairs (.73) than the DZ twin pairs (.56), which suggests a role for genetic effects that was not significant given the small sample size in this study. However, a more powerful analysis (DF extremes analysis; DeFries & Fulker, 1985), which treats mathematical skills as a continuous variable rather than as a dichotomy (mathematics disorder/control), yielded a group heritability estimate of .38. This estimate suggests that 38% of the average difference between the twins with mathematics disorder and the unselected population was due to genetic factors.

In a recent study with a much larger sample size (Oliver et al., 2004) the heritability of mathematics disorder was assessed by selecting children in the bottom 15% of the population on teacher ratings of children's mathematical abilities. A strength of this study is the very large sample size (2178 twin pairs), though arguably a weakness is that the teacher ratings of mathematical skills are a less than ideal measure. This

study yielded quite a high group heritability estimate for mathematics disorder of .65 (compared to the estimate of .38 reported by Alarcon et al., 1997). This same study also yielded a similarly sized individual differences heritability estimate of .66 for a composite teacher rating of mathematical skill.

A further study, based on the same Twins Early Development Study (TEDS) sample, used objective Web-based measures of different aspects of mathematical skill: mathematical application, understanding number, computation and knowledge, mathematical interpretation, and non-numerical processes (Kovas, Petrill, & Plomin, 2007). In this study some 2052 children (470 pairs of MZ twins and 781 pairs of DZ twins) were tested when they were 10 years old. This study yielded moderate estimates of heritability for the different aspects of mathematical ability assessed (ranging from .30 to .45). These heritability estimates are lower than the estimate reported by Oliver et al. (.66) and it seems plausible that the lower estimates of genetic influence here may reflect the use of objective measures (rather than teacher ratings, which may be biased because teachers tend to overestimate the degree of similarity in MZ twin pairs). The results from this study might be seen as supporting the importance of generalist genes as influences on diverse aspects of mathematical ability (as advocated by Plomin & Kovas, 2005).

In summary, current evidence suggests that there are substantial genetic influences on mathematics disorder and it has been argued that the same genetic influences may also operate to influence individual differences among people in the normal range of mathematical ability (Plomin & Kovas, 2005). However, even accepting such heritability estimates, there remains room for substantial environmental influences on mathematical skills. Furthermore, the heritability for the mathematics disorder group deficit should not be taken to imply that remedial teaching or other interventions cannot be effective in helping to improve those children's mathematical skills.

There are no "genes" for mathematics or for mathematics disorder. However, genes do affect processes controlling protein synthesis, which, via the processes operating in epigenesis, affect the development of the brain structures that allow us to learn mathematics. For these reasons it is important not to see genetic effects as deterministic and immutable. Instead, genes operate in the context of a wide range of biological and experiential factors to influence the development of our ability to learn and to perform mathematics.

Brain bases of mathematics disorder

Until recently the vast majority of work on the brain bases of arithmetic and its disorders has been with adults, though recently work has begun to examine brain mechanisms in children with mathematics disorder. We will begin by considering work on adults as a foundation for the smaller amount of work with children.

Based on a review of brain imaging and the effects of brain lesions in adults Dehaene, Piazza, Pinel, and Cohen (2003) proposed three separable, though interconnected, brain systems in the parietal lobe that play a role in number processing (see Plate 5). The neural substrate of a "number sense" system that is activated when comparing numerical magnitudes or estimating appears to depend critically upon bilateral areas of the horizontal intraparietal sulcus (HIPS). Bilaterally areas of the posterior superior parietal lobe (PSPL) also appear to be activated in tasks that require the shifting of spatial attention, such as approximating and number comparison tasks, and it was suggested that this system supports a process of orienting attention to particular regions of a mental "number line." Finally, the left angular gyrus (the area that when lesioned gives rise to Gerstmann's syndrome) appears to be more active in verbal calculation tasks such as exact addition. In addition to these three "core" number areas, areas in the prefrontal and cingulate cortex are systematically activated when adults are asked to perform calculations, and these other areas may (speculatively) be involved in attentional processes required in calculation.

It seems reasonable to suppose that problems in the development of the brain systems identified as critical to arithmetic in adults may be fundamental to the problems observed in children with mathematics disorder, though as yet direct evidence for this is lacking (Wilson & Dehaene, 2007). Consistent with the idea of arithmetic difficulties being associated with parietal dysfunction, several studies have shown parietal deficits in Turner's syndrome, a syndrome associated with arithmetic deficits (e.g., Reiss, Mazzocco, Greenlaw, Freund, & Ross, 1995). Perhaps most strikingly, Isaacs, Edmonds, Lucas, and Gadian (2001) reported a specific reduction in gray matter in the left HIPS in a group of adolescents with mathematics disorder (without reading disorder) who had been born prematurely, compared to a control group without mathematics disorder who had been born equally prematurely. This difference in the left HIPS was only found for children with problems with calculation, and not for another group of children who had problems with mathematical reasoning.

Further evidence for the role of the HIPS in number processing comes from recent brain imaging studies. Cantlon, Brannon, Carter, and Pelphrey (2006) used a numerosity adaptation paradigm (see Figure 5.7) in which subjects view a series of displays of the same number of items that differ in other respects (size and shape). They found that changes in numerosity (but not changes in irrelevant attributes such as shape) resulted in increased activation (measured by fMRI) in the HIPS in 4-year-old children as well as adults. This study suggests that the approximate numerical system of preschool children has structural and functional similarities with the numerical system used by adults. The finding that the HIPS is active during this task as well as during number comparison judgment tasks (deciding which digit represents the larger magnitude – 3 vs. 7) suggests that symbolic number representation in adults may build upon an approximate number sense system, with both depending upon neural systems in the HIPS. Temple and Posner (1998) investigated brain potentials during symbolic and nonsymbolic number comparison in adults and 5-year-old children and found that ERP localization in children was similar to adults. This again is consistent with the idea that comparisons of numerical and physical magnitudes depend upon common neural mechanisms in both adults and children.

As far as we are aware only one study has compared patterns of brain activation in children with mathematics disorder and in matched typically developing children (Kucian et al., 2006). In this study children completed three tasks: approximate calculation, exact calculation, and nonsymbolic magnitude comparison (see Box 5.3).

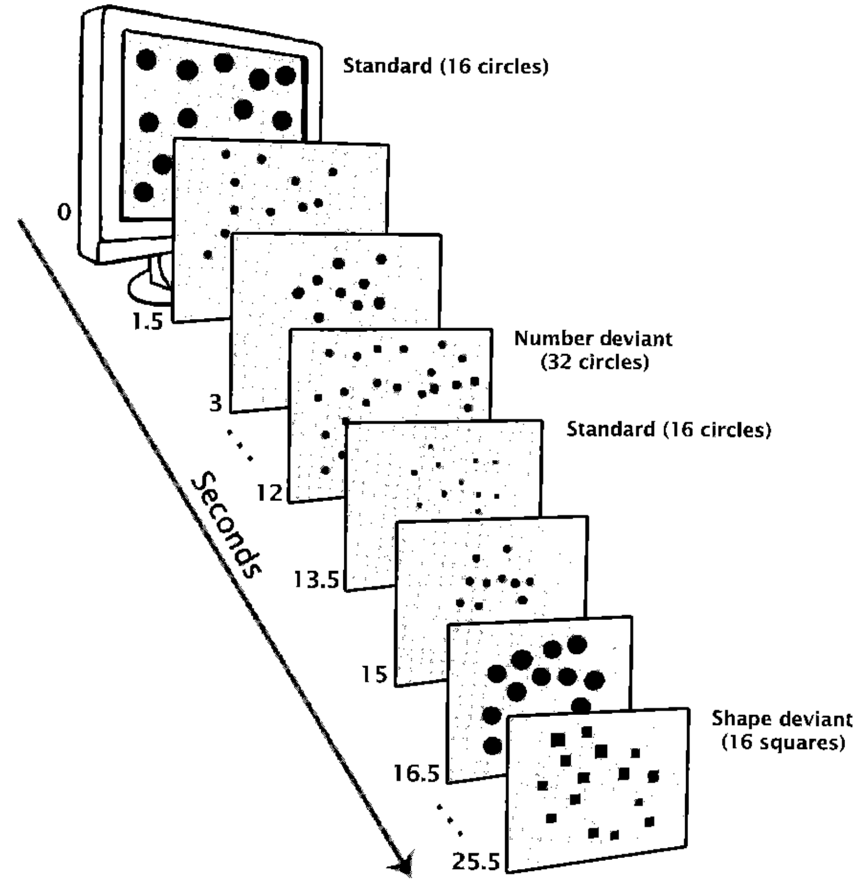


Figure 5.7 The numerosity adaptation task used by Cantlon et al. (2006). Here the participants (adults and 4-year-old children) passively view a stream of displays on a computer screen. The majority of displays contain the same number and same type of elements (16 circles of differing size). Occasionally a display is presented that deviates from the standard *either* in the number of elements present (Number deviants) *or* the shape of the elements (Shape deviants). (Adapted from Cantlon, Brannon, Carter, & Pelphrey, 2006.)

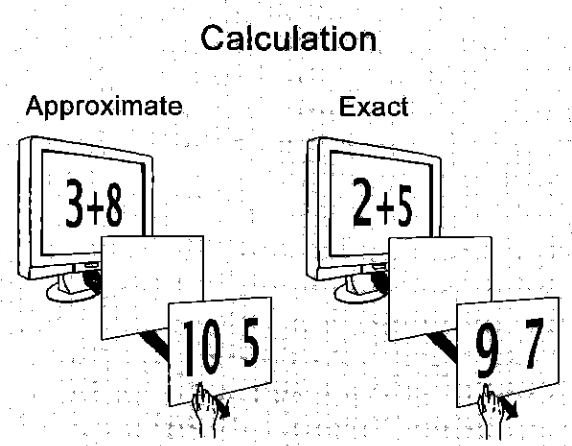
There were no significant behavioral differences in performance in these tasks between children with and without mathematics disorder. During the exact calculation and magnitude comparison tasks children with mathematics disorder also activated similar parietal and prefrontal regions to children in the control group. However, during approximate calculation, children with mathematics disorder showed less parietal activation than control children. This could be interpreted as evidence for a missing or less developed link between the approximate numerical system and symbolic number representation system in children with mathematics disorder (described in this study as dyscalculia). Caution is needed here, as this is the first study of its kind and activations in those areas were positively correlated with accuracy.

The brain activation patterns of children with mathematics disorder ($N = 18$) and control children ($N = 20$) during each condition are shown in Plate 6. Children with mathematics disorder (children with DD or developmental dyscalculia in Kucian et al.'s terminology) showed greater variability among children and had weaker activation in most of the neuronal network involved in approximate calculation, including the intraparietal sulcus and the middle and inferior frontal gyrus of both hemispheres. There was evidence that the left intraparietal sulcus, the left inferior frontal gyrus, and the right middle frontal gyrus seemed to play crucial roles in correct approximate calculation because brain activation correlated with accuracy in approximate calculation in these regions.

Box 5.3 Approximate calculation, exact calculation, and magnitude comparison tasks (Reprinted with permission from Kucian, K., Loenneker, T., Dietrich, T., Dosch, M, Martin E., and von Aster, M. G., *Behavioral and Brain Functions*, 2 (31), 2006)

The tasks performed under fMRI consisted of approximate calculation and exact calculation, approximate and exact control conditions, and a magnitude comparison task.

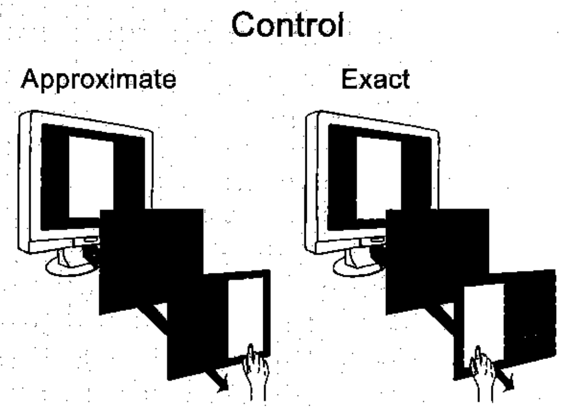
Calculation task
The calculation task consisted of three cycles of alternating approximate and exact calculation blocks. In the approximate calculation task the child selects the number that is closest to the correct answer to the sum. In the exact calculation task the child selects the number that corresponds to the correct answer.



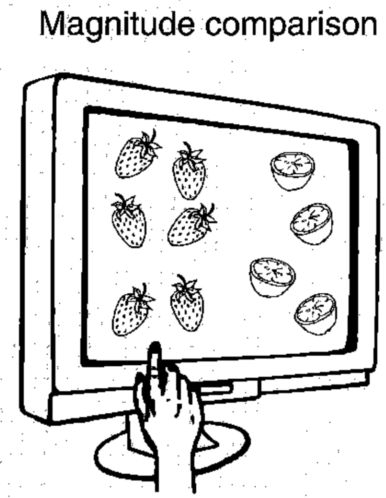
Control task: Luminosity
The control condition for the calculation trials was a discrimination task involving gray light patches, again presented during three cycles of approximate and exact discrimination blocks. In the exact control task, subjects had to

Box 5.3 (*cont'd*)

match sequentially presented gray-scale patterns. In the approximate control task, they were asked to pick the gray-scale pattern with the most similar luminosity (brightness) to the standard. Alternative solutions were more alike in the exact control condition than those in the approximate control condition.



Magnitude comparison
In the magnitude comparison task participants had to compare two sets of different objects (pictures of fruit or vegetables) and select the set with the larger number of objects. The maximum number of objects displayed on one side was 18. The differences between the two sets were: 1, 2, 3, or 4 in the first block; 9, 10, 11, or 12 in the second block; and 5, 6, 7, or 8 in the third block. Fixation during rest served as the control condition for magnitude comparison.



Summary of brain bases of mathematics disorder

There is evidence for neural circuits involved in the understanding of physical and numerical magnitudes and calculation that include the horizontal intraparietal sulcus (HIPS), the posterior superior parietal lobe (PSPL), the left angular gyrus, and areas in the prefrontal and cingulate cortex. Current evidence suggests that underdevelopment of the HIPS may result in a deficient "number sense" system, and that such problems are associated with mathematics disorder at least in some cases (Isaacs et al., 2001). Children with mathematics disorder also appear to show less activation in parietal areas (which include, but are broader than, the HIPS) during approximate calculation tasks.

Interventions to Improve Mathematics

There are a small number of good quality studies that have sought to improve the mathematical skills of young children considered to be at risk of developing mathematics disorder. So far, however, there are no studies we are aware of that have investigated the effectiveness of interventions for older children who have developed mathematics disorder. This is clearly an important area for further research (Dowker, 2005).

Prevention

Griffin and colleagues (Griffin & Case, 1996; Griffin, Case, & Siegler, 1994) conducted a whole-class intervention study using "Number Worlds": a package of teacher-led whole-class instruction, interactive games, and other activities designed to improve the number skills of disadvantaged kindergarten children. This was a theoretically based intervention designed to facilitate the development of a "number line representation" in these children. At the end of the kindergarten year the intervention group performed better than an untreated control group and their attainments in conceptual and procedural tests of arithmetic approached those of a normative comparison group. This is an encouraging result, though it does not mean that such a program will prevent the development of mathematics disorder in the small minority of children at risk of the disorder.

Ramani and Siegler (2008) reported encouraging results from a short-term intervention study conducted with 5-year-old disadvantaged children attending a head-start program in the USA. The intervention was based on children playing a board game with an adult. The "number" board game had 10 consecutively numbered squares on the board, and the child span a spinner that showed a 1 or a 2. The child and the adult took turns in the game and moved their place holder on the board in accordance with the number shown on the spinner. Children were required to count on from the number on the square where they were. So, for example, if the child was on square 3 and the spinner indicated a 2 they would say "4, 5" as they moved their place holder on the board to square 5 (the numbers were also marked on the squares of the board). An analogous form of the board game involved only colored squares without numbers and a spinner that had colors on it corresponding to the colors of the squares on the board. Children were assigned randomly to the number or color version of the game, and played it in individual sessions with an adult for five sessions lasting roughly 20 min each. Before and after the intervention, numerical skills were assessed with four tasks: counting from 1 to 10; number line estimation (marking a line marked with 0 and 10 at the ends to indicate the position of an intermediate number); numerical magnitude comparison (choosing the numerically large number from two numbers presented side by side); and numeral identification (naming a series of numerals between 1 and 10 presented in random order). The children in the number version of the game showed substantial improvements in numeral identification ($d = .69$), numerical magnitude comparison ($d = .79$), counting ($d = .74$), number line estimation accuracy ($d = .76$), and the linearity of number line judgments ($d = 1.00$). In a sense, improvements in numeral identification and counting measures are unsurprising since they were practised during the game. However, it is encouraging that there were also gains in numerical magnitude comparison and number line estimation, two tasks that arguably are less directly targeted in the game and might be seen as tapping children's understanding of numerical quantities. The effects obtained in this study (with young typically developing children) are encouraging, given the brief time spent on the "game-based" intervention.

Fuchs et al. (2005) conducted a large-scale intervention study with 1st grade children identified as being at risk of developing mathematics disorder. Based on screening 564 children from 41 1st grade classes, 127 children were identified as being at risk of developing mathematics disorder based on a group test of mathematics, followed by individual testing of over 300 children with the Woodcock-Johnson III (Woodcock, McGrew, & Mather, 2001) Calculation and Applied Problems mathematics test. These at-risk children represented roughly the bottom 21% of the sample in terms of their mathematical skills and they were randomly assigned to an intervention (70) or a nonintervention (69) group. The maths outcomes for these two groups were compared to those of a not-at-risk group of 180 children. The at-risk groups had full-scale IQ scores of 85, compared to 95 for the not-at-risk group.

The intervention was delivered by 12 trained tutors to 37 small groups of two or three children in 30-min sessions. Each group teaching session was followed by 10-min individual sessions with a computer program that gave simple addition and subtraction problems for the child to solve, coupled with feedback on correct responses. A total of 48 teaching sessions were given. The intervention sessions followed a highly structured, scripted format and the groups only moved on to harder activities when all children showed mastery of the concepts taught in a session (based on an individually administered test at the end of each session). Hence the program, though taught to groups of two or three children at a time, was sensitive to variations in children's rate of learning. The trained at-risk children showed significantly greater improvements than the untreated at-risk group (with medium effect sizes ranging from $d = 0.57$ to 0.70) on three measures: Woodcock-Johnson

Calculation (a written test of basic arithmetic), Grade 1 Concepts Applications (a standardized spoken test with written responses tapping a variety of mathematical concepts), and Story Problems (a set of arithmetic problems presented orally in story format). However there were no significant improvements between these two groups on the other four outcome measures tapping mathematical skills.

Estimates of how many children would satisfy a conventional discrepancy-based criterion for mathematics disorder (a discrepancy of 1 SD between IQ and math) using a variety of outcome measures showed average rates of around 3.2% for the at-risk untreated group compared to around 2.5% for the at-risk treated group. It was found that teachers' ratings of children's attention, their working memory scores (on a listening span measure), and phonological awareness scores were all unique predictors of some of the math outcome measures after the effects of group, intervention, and a range of other possible predictors had been controlled.

Wilson, Revkin, Cohen, Cohen, and Dehaene (2006) reported the results of a small-scale intervention study with nine 7–9-year-old children with mathematics disorder. The children were given a short-term intensive intervention involving a computer-based game designed to improve their "number sense." The results showed improvements after the intervention on some measures (subitizing speed and numerical magnitude comparison speed) but not on other measures, some of which are arguably closer to everyday arithmetic skills (e.g., addition speed). In addition the absence of a control group makes it difficult to draw any strong conclusions from this study since we do not know how much improvement the children may have made on the speeded number judgment measures simply as a result of repeated testing.

Summary of interventions for mathematics

The evidence from these prevention and intervention studies is encouraging insofar as they show positive effects on children's arithmetic skills, and in one case a reduction in the rates of children who would qualify for a diagnosis of mathematics disorder. The effects obtained by Fuchs et al. (2005) were, however, quite variable across the measures used (with null effects on a number of measures). It appears that we are in need of further large-scale studies of this sort that preferably follow children for longer periods of time. So far we really do not have any evidence concerning how well older children with severe mathematics disorder can be helped to overcome their problems and this is an area where research is badly needed.

It is worth noting here that problems of anxiety specifically related to math are common in adults, and furthermore it appears that such math anxiety operates specifically to interfere with the working memory operations needed to solve more complex math problems (Ashcraft, Kirk, & Hopko, 1998). In this view math anxiety can have clear adverse effects on performing, and presumably on learning to perform, arithmetic. As far as we know comparable studies have not been conducted with children and in particular no studies of math anxiety in children with mathematics disorder have been reported. However, the developmental implications of this research are clear. It seems likely that early difficulties in learning math may contribute

to the development of math anxiety, which in turn will impede the processes involved in performing and learning math. This suggests that early interventions to help circumvent math problems, and the anxiety associated with them, may be particularly valuable.

Summary and Conclusions

Mathematics disorder (problems in mastering number skills and arithmetic) is relatively common in children. There are some clear analogies between mathematics disorder and dyslexia insofar as both of these conditions affect relatively circumscribed areas of cognition that have very direct educational implications. Compared to studies of dyslexia, however, our understanding of mathematics disorder remains quite limited. This reflects a less advanced understanding of typical arithmetic development, compared to typical reading development, and also the fact that much less research has directly focused on children with mathematics disorder than on children with dyslexia. Research on arithmetic development and mathematics disorder now appears to be increasing rapidly.

A number of clear conclusions can be drawn from the work we have considered in this chapter. Though pure cases of Mathematics Disorder occur, it is important to emphasize that many more children have mathematics disorder and reading disorder. These MD/RD children typically have more severe arithmetic problems than children with mathematics disorder alone. It seems likely that this is because the pure MD children have a more limited cognitive deficit than children with MD/RD. One hypothesis is that pure mathematics disorder might commonly arise from a deficit in a nonverbal "number sense" system, and that MD/RD children have additional difficulties with the verbal aspects of learning arithmetic. Understanding the similarities and differences between children with mathematics disorder and mathematics disorder/reading disorder is a key issue for future research.

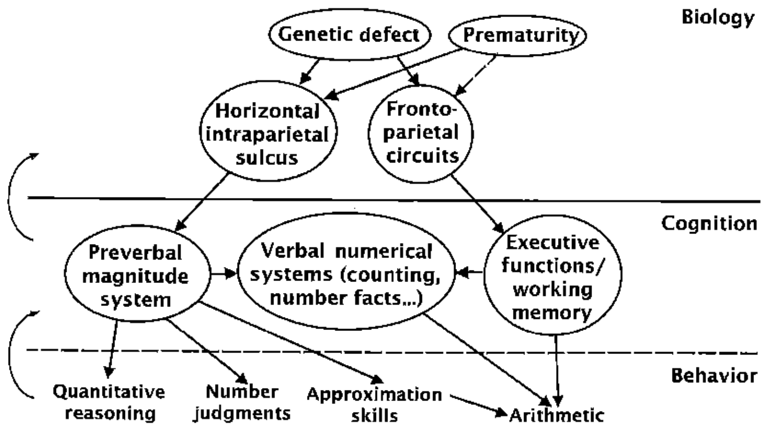


Figure 5.8 A path model of mathematics disorder.

A better understanding of the cognitive bases of mathematics disorder will require longitudinal studies to identify the likely causes of the disorder, and it will be important to try to use relatively pure measures of arithmetic rather than standardized tests that typically conflate many different skills (such as addition, subtraction, and multiplication) into a single score. As in the case of reading disorders, impressive advances have been made in understanding the genetic and brain mechanisms of mathematics disorder. Multiple overlapping sets of genes seem to be responsible for influencing normal variations in arithmetic and the arithmetic deficits seen in mathematics disorder. It seems likely that these genes will influence the development of brain structures in the parietal lobe that underlie our ability to understand numerical magnitudes and perform calculations. However, it is also likely that genetic influences will operate by influencing the development of other brain regions that have also been identified as being involved in learning and performing arithmetic. Figure 5.8 shows a causal path model of mathematics disorder that is consistent with the major findings we have considered in this chapter.

6

Developmental Coordination Disorder

Problems with the development of motor skills in children are relatively common and may be associated with a number of conditions that affect the brain and nervous system (such as cerebral palsy). Here we will focus on children who experience problems in developing motor skills although they do not suffer from any diagnosed disease. Such problems are referred to in DSM-IV as developmental coordination disorder (DCD). Older terms that have been used to refer to this group of children include clumsy child syndrome and developmental dyspraxia and agnosia (agnosia literally means a difficulty in recognizing objects; and this terminology stresses that these children typically have difficulties affecting both motor control and perception).

Definitions and Prevalence

In DSM-IV the criteria for diagnosing DCD are significant problems in motor coordination that are out of line with those expected for a child's age and IQ. It is specified that such problems should significantly interfere with a child's academic achievements (problems with handwriting and drawing are common) or with activities in daily life (problems in learning to dress, and in sports and games, are common). This is a discrepancy definition, analogous to the definitions of other forms of specific learning difficulty, because problems with motor skills need to be out of line with IQ (taking account of the fact that severe learning difficulties may be associated with motor coordination problems). The definition is also a developmental one because the assessment of motor coordination difficulties needs to be related to a child's age. The usefulness of using IQ as a means of excluding children from getting a diagnosis of DCD has been questioned (Geuze, Jongmans, Schoemaker, & Smits-Englesman, 2001; Henderson & Barnett, 1998). There is a lack of good evidence for a correlation between IQ and motor skills (at least in the normal range) and it is clear that some children with severe learning difficulties (IQ below 70) perform adequately on motor tasks. It has been suggested, therefore, that it may be more useful to consider cases of children with low IQ and poor motor skills as showing DCD with comorbid learning problems (Geuze et al., 2001).